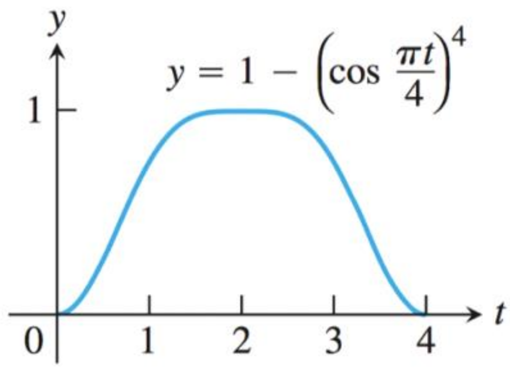
Problem Solving (Template)

7 Aralık 2020 Pazartesi 22:42

Average Value of a Function

In Exercises 15–18, use a finite sum to estimate the average value of f on the given interval by partitioning the interval into four subintervals of equal length and evaluating f at the subinterval midpoints.

$$f(t) = 1 - \left(\cos \frac{\pi t}{4}\right)^4 \quad \text{on} \quad [0, 4]$$



21. Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of the polygon for the following values of n:

a. 4 (square) **b.** 8 (octagon) **c.** 16

d. Compare the areas in parts (a), (b), and (c) with the area of the circle.

22. (*Continuation of Exercise 21.*)

a. Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of one of the n congruent triangles formed by drawing radii to the vertices of the polygon.

b. Compute the limit of the area of the inscribed polygon as $n \to \infty$.

c. Repeat the computations in parts (a) and (b) for a circle of radius r.

Evaluate the sums

25. $\sum_{k=1}^{5} k(3k + 5)$

26. $\sum_{k=1}^{7} k(2k + 1)$

27. $\sum_{k=1}^{5} \frac{k^3}{225} + \left(\sum_{k=1}^{5} k\right)^3$

28. $\left(\sum_{k=1}^{7} k\right)^2 - \sum_{k=1}^{7} \frac{k^3}{4}$

29. **a.** $\sum_{k=1}^{7} 3$ **b.** $\sum_{k=1}^{500} 7$ **c.** $\sum_{k=3}^{264} 10$

30. **a.** $\sum_{k=9}^{36} k$ **b.** $\sum_{k=3}^{17} k^2$ **c.** $\sum_{k=18}^{71} k(k - 1)$

31. **a.** $\sum_{k=1}^{n} 4$ **b.** $\sum_{k=1}^{n} c$ **c.** $\sum_{k=1}^{n} (k - 1)$

32. **a.** $\sum_{k=1}^{n} \left(\frac{1}{n} + 2n\right)$ **b.** $\sum_{k=1}^{n} \frac{c}{n}$ **c.** $\sum_{k=1}^{n} \frac{k}{n^2}$

Limits of Riemann Sums

For the functions in Exercises 39–46, find a formula for the Riemann sum obtained by dividing the interval $[a, b]$ into n equal subintervals and using the right-hand endpoint for each c_k. Then take a limit of these sums as $n \to \infty$ to calculate the area under the curve over $[a, b]$.

39. $f(x) = 1 - x^2$ over the interval $[0, 1]$.

40. $f(x) = 2x$ over the interval $[0, 3]$.

41. $f(x) = x^2 + 1$ over the interval $[0, 3]$.

42. $f(x) = 3x^2$ over the interval $[0, 1]$.

43. $f(x) = x + x^2$ over the interval $[0, 1]$.

44. $f(x) = 3x + 2x^2$ over the interval $[0, 1]$.

Using Known Areas to Find Integrals

In Exercises 15–22, graph the integrands and use areas to evaluate the integrals.

15. $\displaystyle\int_{-2}^{4}\left(\frac{x}{2} + 3\right) dx$

16. $\displaystyle\int_{1/2}^{3/2}(-2x + 4)\, dx$

17. $\displaystyle\int_{-3}^{3}\sqrt{9 - x^2}\, dx$

18. $\displaystyle\int_{-4}^{0}\sqrt{16 - x^2}\, dx$

19. $\displaystyle\int_{-2}^{1}|x|\, dx$

20. $\displaystyle\int_{-1}^{1}(1 - |x|)\, dx$

21. $\displaystyle\int_{-1}^{1}(2 - |x|)\, dx$

22. $\displaystyle\int_{-1}^{1}\left(1 + \sqrt{1 - x^2}\right) dx$

Average Value of a Continuous Function Revisited

DEFINITION If f is integrable on $[a, b]$, then its **average value on [*a*, *b*]**, also called its **mean**, is

$$\text{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Finding Average Value

In Exercises 55–62, graph the function and find its average value over the given interval.

55. $f(x) = x^2 - 1$ on $\left[0, \sqrt{3}\right]$

56. $f(x) = -\frac{x^2}{2}$ on $[0, 3]$

57. $f(x) = -3x^2 - 1$ on $[0, 1]$

58. $f(x) = 3x^2 - 3$ on $[0, 1]$

59. $f(t) = (t - 1)^2$ on $[0, 3]$

60. $f(t) = t^2 - t$ on $[-2, 1]$

61. $g(x) = |x| - 1$ on **a.** $[-1, 1]$, **b.** $[1, 3]$, and **c.** $[-1, 3]$

62. $h(x) = -|x|$ on **a.** $[-1, 0]$, **b.** $[0, 1]$, and **c.** $[-1, 1]$

What values of a and b minimize the value of

$$\int_a^b (x^4 - 2x^2)\, dx?$$