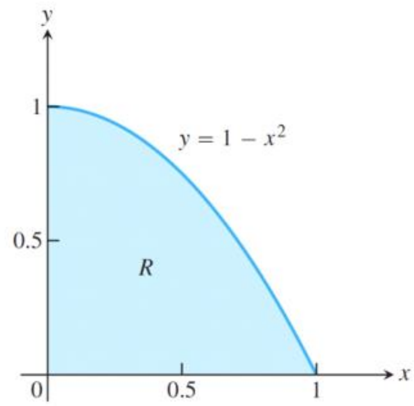
Lecture_Template

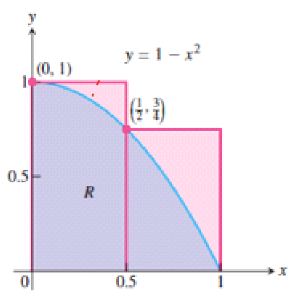
7 Aralık 2020 Pazartesi 00:16

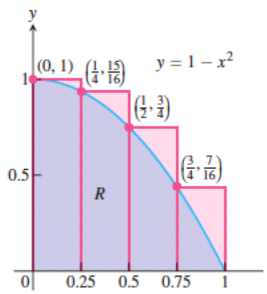
5 INTEGRATION

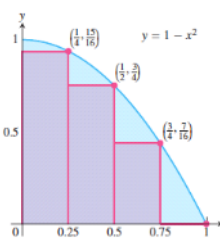
5.1 Area and Estimating with Finite Sums

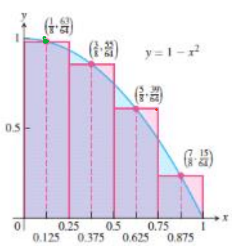
If you don't know anything about "integral", how can you calculate exact area of R?

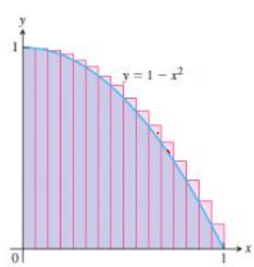


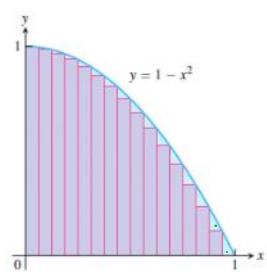












Area

In Exercises 1–4, use finite approximations to estimate the area under the graph of the function using

a. a lower sum with two rectangles of equal width.
b. a lower sum with four rectangles of equal width.
c. an upper sum with two rectangles of equal width.
d. an upper sum with four rectangles of equal width.

$f(x) = 4 - x^2$ between $x = -2$ and $x = 2$.

Using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base (*the midpoint rule*), estimate the area under the graphs of the following functions, using first two and then four rectangles.

$f(x) = 1/x$ between $x = 1$ and $x = 5$.

Average Value of a Nonnegative Continuous Function

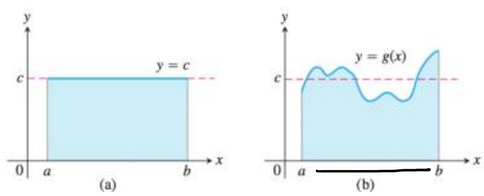


FIGURE 5.6 (a) The average value of $f(x) = c$ on $[a, b]$ is the area of the rectangle divided by $b - a$. (b) The average value of $g(x)$ on $[a, b]$ is the area beneath its graph divided by $b - a$.

Ex: Use a finite sum to estimate the average value of f on the given interval into four subintervals of equal length and evaluating f at the subinterval midpoints.

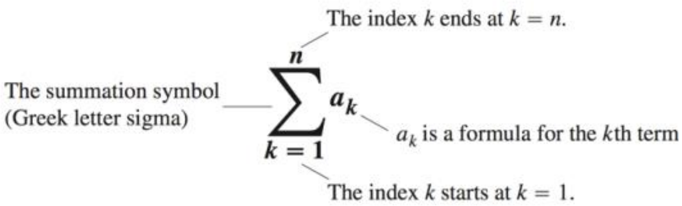
$$f(x) = \frac{4}{x} \text{ on } [2,18]$$

5.2 Sigma Notation and Limits of Finite Sums

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n.$$

The index k ends at $k = n$.

The summation symbol (Greek letter sigma) $\sum^{n} a_k$



Algebra Rules for Finite Sums

1. *Sum Rule:* $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$
2. *Difference Rule:* $\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$
3. *Constant Multiple Rule:* $\sum_{k=1}^{n} ca_k = c \cdot \sum_{k=1}^{n} a_k$ (Any number c)
4. *Constant Value Rule:* $\sum_{k=1}^{n} c = n \cdot c$ (c is any constant value.)

Ex: Which of the following express $1 - 2 + 4 - 8 + 16 - 32$ in sigma notation?

a. $\sum_{k=1}^{6}(-2)^{k-1}$ **b.** $\sum_{k=0}^{5}(-1)^{k} 2^{k}$ **c.** $\sum_{k=-2}^{3}(-1)^{k+1} 2^{k+2}$

Ex: $\sum_{k=-3}^{-1} k^2 =$

Ex: $-\frac{1}{5} + \frac{2}{5} - \frac{3}{5} + \frac{4}{5} - \frac{5}{5}$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Ex: $\sum_{k=0}^{5} k^3 =$

$\sum_{k=3}^{15} k^3 =$

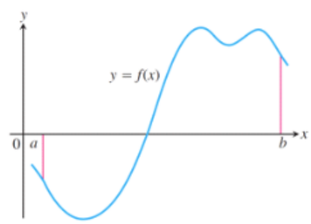
Ex: $\sum_{k=3}^{8} 4 =$

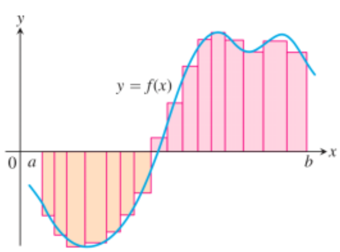
Ex: $\sum_{k=1}^{7} k(2k+1) =$

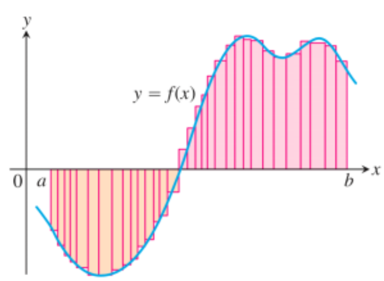
Riemann Sum

$$S_P = \sum_{k=1}^{n} f(c_k)\,\Delta x_k .$$

Riemann sum for *f* on the interval [*a*, *b*]



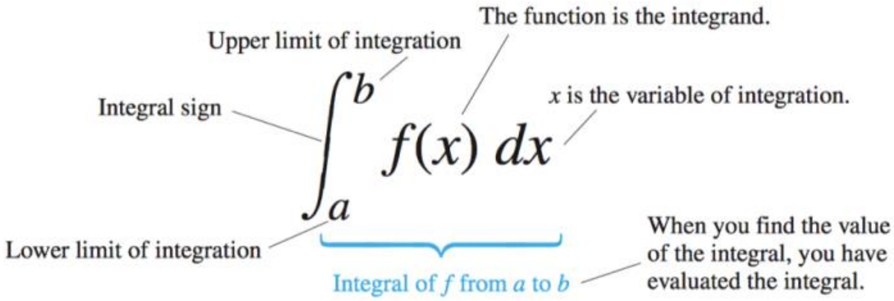




$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

Ex: Find $\int^5 (x^2+2)\,dx$ by Riemann sum.

5.3 The Definite Integral



Integrable and Nonintegrable Functions

THEOREM 1—Integrability of Continuous Functions If a function f is continuous over the interval $[a, b]$, or if f has at most finitely many jump discontinuities there, then the definite integral $\int_a^b f(x)\,dx$ exists and f is integrable over $[a, b]$.

TABLE 5.4 Rules satisfied by definite integrals

1.	*Order of Integration:*	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	A Definition
2.	*Zero Width Interval:*	$\int_a^a f(x)\,dx = 0$	A Definition when $f(a)$ exists
3.	*Constant Multiple:*	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$	Any constant k
4.	*Sum and Difference:*	$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$	
5.	*Additivity:*	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$	
6.	*Max-Min Inequality:*	If f has maximum value $\max f$ and minimum value $\min f$ on $[a, b]$, then $\min f \cdot (b - a) \le \int_a^b f(x)\,dx \le \max f \cdot (b - a).$	
7.	*Domination:*	$f(x) \ge g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$	
		$f(x) \ge 0$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge 0$	(Special Case)

Suppose that $\int_1^2 f(x)\,dx = 5$. Find

a. $\int_1^2 f(u)\,du$ **b.** $\int_1^2 \sqrt{3} f(z)\,dz$

c. $\int_2^1 f(t)\,dt$ **d.** $\int_1^2 [-f(x)]\,dx$

Use the Max-Min Inequality to find upper and lower bounds for the value of

$$\int_0^1 \frac{1}{1+x^2}\,dx.$$

Area Under the Graph of a Nonnegative Function

DEFINITION If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ over $[a, b]$** is the integral of f from a to b,

$$A = \int_a^b f(x)\,dx.$$

Average Value of a Continuous Function Revisited

DEFINITION If f is integrable on $[a, b]$, then its **average value on $[a, b]$**, also called its **mean**, is

$$\text{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

$$f(x) = -\frac{x^2}{2} \quad \text{on} \quad [0, 3]$$