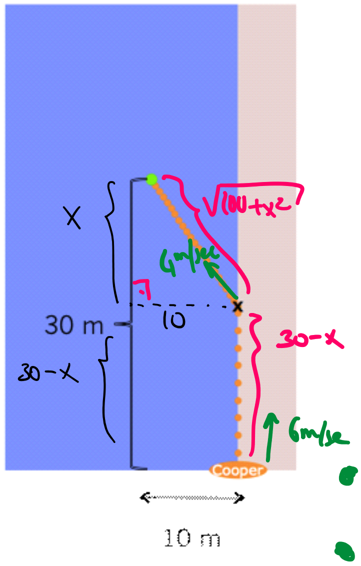
Problem Solving_sec03

29 Kasım 2020 Pazar 19:45

Ex:



We have a dog named Cooper.

Cooper is on the beach,
we throw a tennis ball towards to sea and
it tries to catch it.
Cooper can run on the beach with velocity
6 m/sec and it can swim in water with
velocity 4 m/sec.

If we throw the ball 30 m far up and
10 m far into the sea, where Cooper
should jump into the sea to catch the ball
in a minimum time.

$$t(x) = \underbrace{t_{\text{spend on the beach}}}_{\frac{30-x}{6}} + \underbrace{t_{\text{spend in the sea}}}_{\frac{\sqrt{100+x^2}}{4}}$$

$$t(x) = \frac{30-x}{6} + \frac{\sqrt{100+x^2}}{4}, \quad 0 \le x \le 30$$

$$\frac{30-x}{6} = \frac{30}{6} - \frac{x}{6}$$

$$t'(x) = -\frac{1}{6} + \frac{1}{4}\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{100+x^2}}$$

$$t'(x) = \frac{3x - 2\sqrt{x^2+100}}{12\sqrt{x^2+100}}$$

$t' = 0 \Rightarrow$

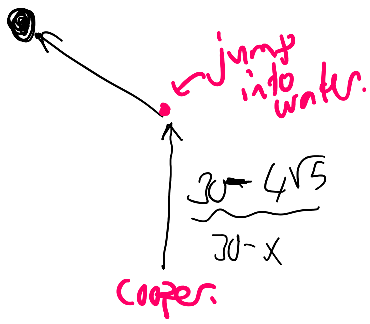
$$3x = 2\sqrt{x^2+100}$$
$$9x^2 = 4(x^2+100)$$
$$5x^2 = 400$$
$$x^2 = 80 \quad \boxed{x = 4\sqrt{5}} \quad \cancel{x = -4\sqrt{5}}$$

cr. pt

$t' = \text{undef} \Rightarrow$ no pt!

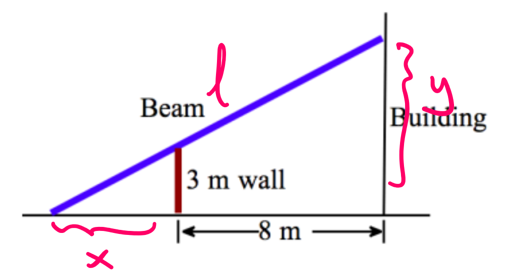
	0		$4\sqrt{5}$		30
t'		−	0	+	
t		↘		↗	

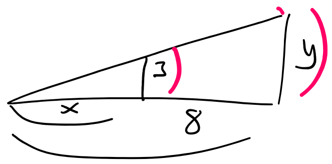
$x = 4\sqrt{5} \Rightarrow t$ attains its min!



4.6.45

The 3-m wall shown here stands 8 m from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



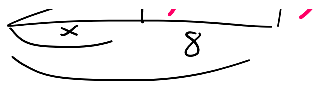


$$l^2 = (x+8)^2 + y^2$$

$$\frac{x}{x+8} = \frac{3}{y} \Rightarrow \boxed{y = \frac{3(x+8)}{x}}$$

$$l^2 = (x+8)^2 + \left(\frac{3(x+8)}{x}\right)^2$$

$$l^2 = (x+8)^2 \cdot \left(1 + \frac{9}{x^2}\right)$$



$$\ell^2 = (x+8)^2 \cdot \left(1+\frac{9}{x^2}\right)$$

minimize $\ell \Rightarrow \ell' = 0$

$$2\ell \cdot \ell' = 2(x+8)\cdot\left(1+\frac{9}{x^2}\right) + (x+8)^2 \cdot \left(9 \cdot \frac{-2}{x^3}\right)$$

$$\vdots$$

$$\underbrace{2\ell}_{\neq 0} \cdot \ell' = (x+8)\left(\frac{2x^3-144}{x^3}\right) \Rightarrow \ell' = 0$$

$\ell' = 0$: ~~$x = -8$~~ or $2x^3 = 144 \Rightarrow x^3 = 72 \Rightarrow \boxed{x = 2\sqrt[3]{9}}$

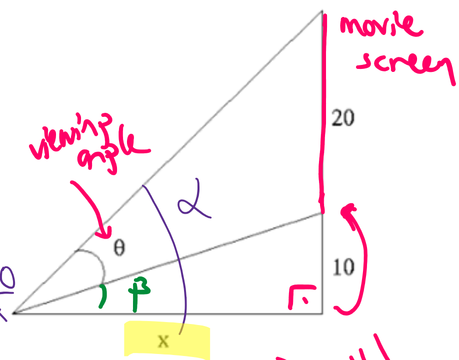
$\ell' = \text{undef}$: ~~$x = 0$~~

	0		$2\sqrt[3]{9}$	
ℓ'		$-$	0	$+$
ℓ		$\searrow$		$\nearrow$

$x = 2\sqrt[3]{9} \Rightarrow \ell$ attains its min value!

Ex: A movie screen on a wall is 20 feet high and 10 feet above the floor.

At what distance x from the front of the room should you position yourself so that The viewing angle of the movie screen is as large as possible?



$$\tan\alpha = \frac{30}{x} \Rightarrow \alpha = \arctan\frac{30}{x}$$

$$\tan\beta = \frac{10}{x} \Rightarrow \beta = \arctan\frac{10}{x}$$

Recall!

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$\theta = \alpha - \beta$$

$$\theta(x) = \arctan\frac{30}{x} - \arctan\frac{10}{x}$$

$$\theta'(x) = \frac{\frac{-30}{x^2}}{1+\left(\frac{30}{x}\right)^2} - \frac{\frac{-10}{x^2}}{1+\left(\frac{10}{x}\right)^2}$$

$$= -\frac{30}{} \cdot \frac{\cancel{x^2}}{} + \frac{10}{} \cdot \frac{\cancel{x^2}}{}$$

$$= -\frac{30}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{x^2+900} + \frac{10}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{x^2+100}$$

- $\theta'(x) = \dfrac{6000 - 20x^2}{(x^2+100)(x^2+900)}$

  → $\theta' = 0 \Rightarrow 20x^2 = 6000$, $x^2 = 300$, $\boxed{x = 10\sqrt{3}}$

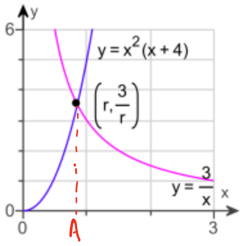
  → θ' = undef → no pts

-

	0		$10\sqrt{3}$	
θ'		$+$	0	$-$
θ		↗		↘

$x = 10\sqrt{3} \Rightarrow \theta$ gets its biggest value!

Ex

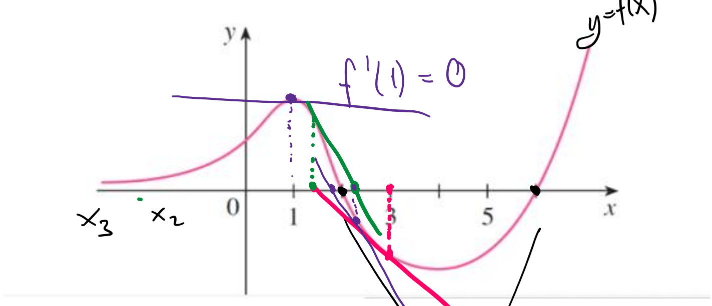


$A \approx ?$ intersection of $y = x^2(x+4)$ and $y = \frac{3}{x}$

Ex: For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown.

(a) $x_1 = 0$ (b) $x_1 = 1$ (c) $x_1 = 3$

(d) $x_1 = 4$ (e) $x_1 = 5$



initial guess ← should be nice.

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(x_n) \neq 0$

a) $x_1 = 0 \rightarrow$ this initial guess is not close to actual root (so not nice) $\Rightarrow$ Newton's Method is not working.

b) $x_1 = 1 \rightarrow f'(1) = 0 \Rightarrow$ Newton's Method won't work

c) $x_1 = 3 \rightarrow$ this is a nice initial guess $\Downarrow$ Newton's Method is working perfectly!

4.7.19

How many solutions does the equation $\sin 2x = 0.22 - 2x^2$ have? Use Newton's method to find them.

a) The equation has 2 solution(s). (Type a whole number.) approximately

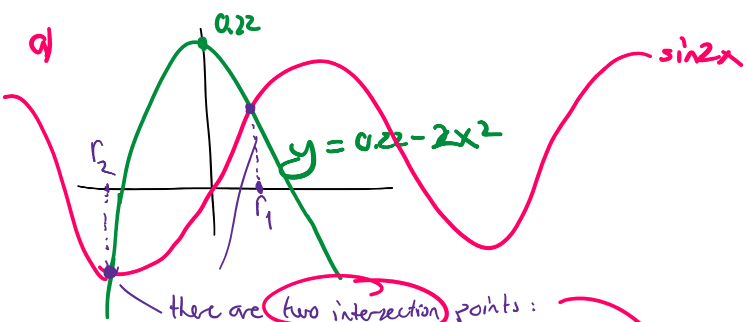
b) The solution(s) is/are x = ☐.

(Use a comma to separate answers as needed. Type an integer or decimal rounded to five decimal places as needed.)

$f(x) = \sin 2x - 0.22 + 2x^2$

$f'(x) = 2\cos 2x + 4x$

a)



there are two intersection points: $\boxed{\sin 2x = 0.22 - 2x^2} \Rightarrow$ have 2 solns

b) Approximation for the roots:

Newton's Method:

root 1 $\Downarrow$ $r_1 \Rightarrow$ $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ first guess

-0.22

$r_1 \Rightarrow \boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$

first guess 0

$n=0$: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{\overset{-0.22}{f(0)}}{\underset{2}{f'(0)}} = +0.11$

$x_2 = \overset{0.11}{x_1} - \frac{f(\overset{0.11}{x_1})}{f'(\underset{0.11}{x_1})} = 0.100622$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 0.100564\ldots$

$\boxed{x_4 = 0.100564\ldots}$ approximation for actual root.

root 2 $\Rightarrow r_2$: $x_0 = -\frac{\pi}{2}$

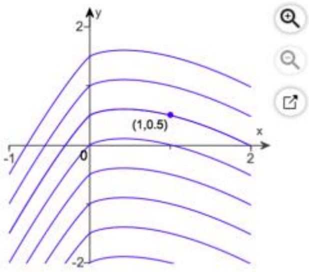
$x_1 = -1.00159$

$x_2 = -0.820269$

$\boxed{x_3 = -0.782814}$ ← approximation root 2 → r_2

4.8.119

The graph below shows solution curves of the differential equation $\frac{dy}{dx} = 1 - \frac{4}{3}x^{1/3}$. Find an equation for the curve that passes through the labeled point.



derivative

$y = f(x) = x - \frac{4}{3}\frac{x^{1/3+1}}{1/3+1} + C$ antiderivative

$\frac{dy}{dx} = f'(x) = 1 - \frac{4}{3}x^{1/3}$

$f(x) = x - \cancel{\frac{4}{3}\frac{x^{4/3}}{4/3}} + C$

$y = f(x) = x - x^{4/3} + C$

$x=1,\ y=0.5 \longrightarrow 0.5 = 1 - 1^{4/3} + C \Rightarrow C = 1/2$

$\Downarrow$

$\boxed{f(x) = x - x^{4/3} + \frac{1}{2}}$

Recall

$(x^n) \xrightarrow{\text{antiderivative}} \left(\frac{x^{n+1}}{n+1}\right)$

derivative

don't!

$\cancel{\frac{n+1\,x^n}{n+1}}$