Lecture_Template

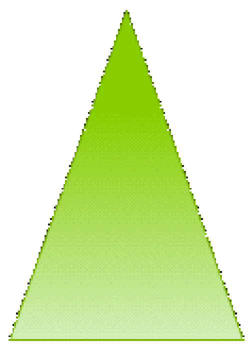
30 Mart 2020 Pazartesi

4.6 Applied Optimization

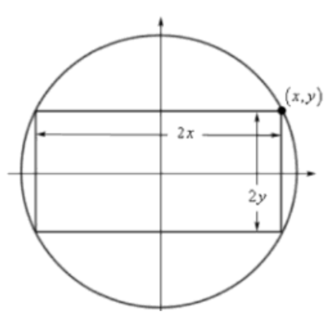
Solving Applied Optimization Problems

1. *Read the problem*. Read the problem until you understand it. What is given? What is the unknown quantity to be optimized?
2. *Draw a picture*. Label any part that may be important to the problem.
3. *Introduce variables*. List every relation in the picture and in the problem as an equation or algebraic expression, and identify the unknown variable.
4. *Write an equation for the unknown quantity*. If you can, express the unknown as a function of a single variable or in two equations in two unknowns. This may require considerable manipulation.
5. *Test the critical points and endpoints in the domain of the unknown*. Use what you know about the shape of the function's graph. Use the first and second derivatives to identify and classify the function's critical points.

Ex: Find the maximum area of an Isosceles triangle which has perimeter 10.



Ex: Determine the area of the largest rectangle that can be inscribed in a circle of radius 4.



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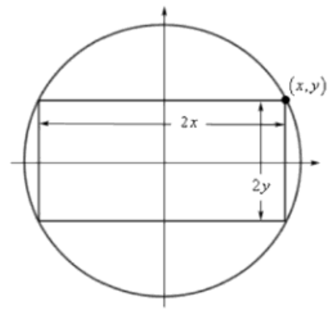
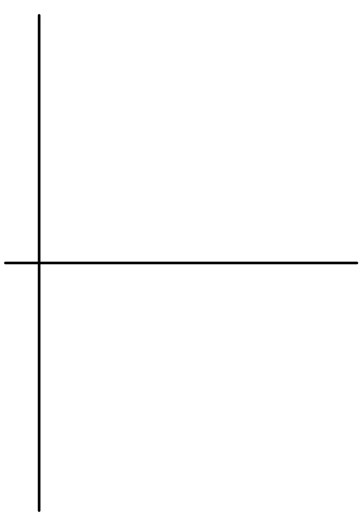


Ex: For $0 \leq t \leq 1$, let $A(t)$ denote the area of the triangle bounded by the x-axis, the y-axis and the tangent line to the curve $y = \ln x$ at $(t, \ln t)$. Find the maximum value of $A(t)$.



4.6 Newton's Method

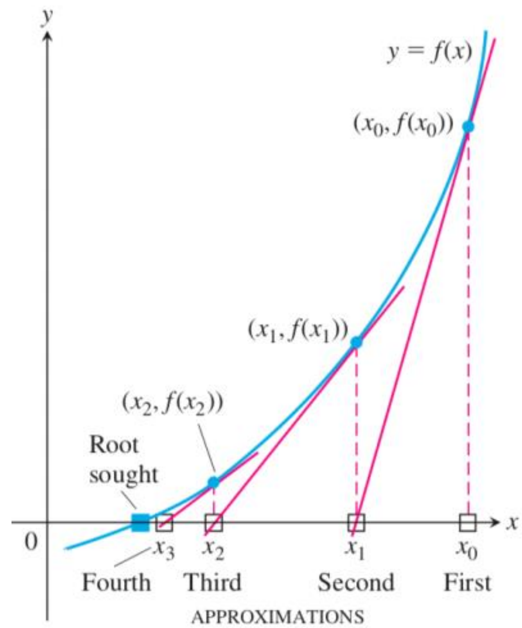
Can you find roots of $10x^3 - 7x^2 - 3x + 1 = \sin x$?

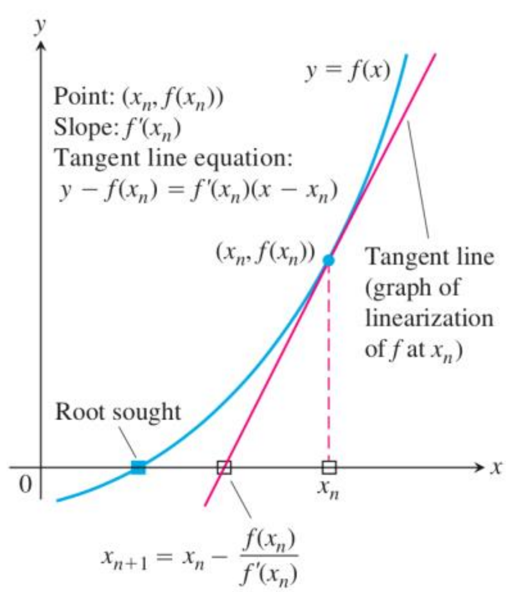
You can not in general, but you can find it approximately.

Newton's Method

1. Guess a first approximation to a solution of the equation $f(x) = 0$. A graph of $y = f(x)$ may help.
2. Use the first approximation to get a second, the second to get a third, and so on, using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad \text{if } f'(x_n) \neq 0. \tag{1}$$



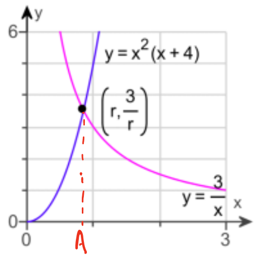


Ex: Starting with $x_0 = -1$ estimate the solution of $x^2 + x - 1 = 0$

Ex: Use Newton's method to find $\sqrt[6]{2}$ correct to four decimals.

Ex: Use Newton's method to find an approximate solution of **ln** (x) = 9 − x. Start with $x_0 = 10$ and find x_2.

Ex

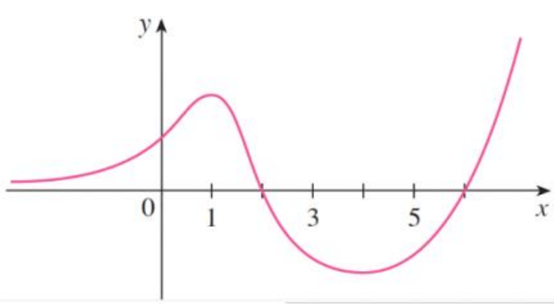


$A \approx ?$ intersection of $y = x^2(x+4)$ and $y = \frac{3}{x}$

Ex: For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown.

(a) $x_1 = 0$ (b) $x_1 = 1$ (c) $x_1 = 3$

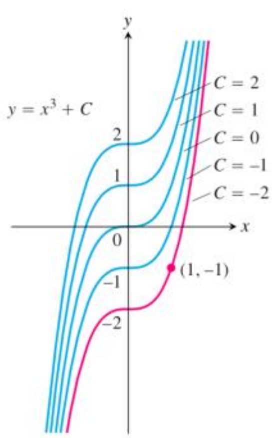
(d) $x_1 = 4$ (e) $x_1 = 5$



4.8 Antiderivatives

DEFINITION A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Ex: What is antiderivative of $3x^2$?



THEOREM 8 If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Ex: $f(x) = \cos 2x$

Ex: $f(x) = 3^x + \frac{1}{\sqrt[3]{x}}$

Ex: $f(x) = -\sec^2 \frac{3x}{2}$

Ex: Find the function with derivative $f'(x) = e^{2x}$ whose graph passes through the point $(0, 3/2)$.

TABLE 4.2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative	Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$	8. e^{kx}	$\frac{1}{k}e^{kx} + C$
2. $\sin kx$	$-\frac{1}{k}\cos kx + C$	9. $\frac{1}{x}$	$\ln\lvert x\rvert + C, \quad x \neq 0$
3. $\cos kx$	$\frac{1}{k}\sin kx + C$	10. $\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$	11. $\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$	12. $\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C, kx > 1$
6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$	13. a^{kx}	$\left(\frac{1}{k\ln a}\right)a^{kx} + C, a > 0, a \neq 1$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$		

TABLE 4.3 Antiderivative linearity rules

	Function	General antiderivative
1. *Constant Multiple Rule:*	$kf(x)$	$kF(x) + C, \quad k$ a constant
2. *Sum or Difference Rule:*	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$

EXAMPLE 4 Find the general antiderivative of

$$f(x) = \frac{3}{\sqrt{x}} + \sin 2x.$$

Indefinite Integrals

DEFINITION The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x, and is denoted by

$$\int f(x)\,dx.$$

The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

EXAMPLE 6 Evaluate

$$\int (x^2 - 2x + 5)\,dx.$$