Problem Solving_Template

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Absolute Extrema on Finite Closed Intervals

In Exercises 21–40, find the absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

28. $h(x) = -3x^{2/3}, \quad -1 \leq x \leq 1$

38. $h(x) = \ln(x + 1), \quad 0 \leq x \leq 3$

39. $f(x) = \dfrac{1}{x} + \ln x, \quad 0.5 \leq x \leq 4$

40. $g(x) = e^{-x^2}, \quad -2 \leq x \leq 1$

In Exercises 45–56, determine critical points and domain endpoints for each function.

53. $y = \ln(x + 1) - \tan^{-1} x$

Local Extrema and Critical Points

In Exercises 57–64, find the critical points and domain endpoints for each function. Then find the value of the function at each of these points and identify extreme values (absolute and local).

63. $y = \begin{cases} -x^2 - 2x + 4, & x \le 1 \\ -x^2 + 6x - 4, & x > 1 \end{cases}$

In Exercises 67–70, show that the function has neither an absolute minimum nor an absolute maximum on its natural domain.

67. $y = x^{11} + x^3 + x - 5$

68. $y = 3x + \tan x$

69. $y = \dfrac{1 - e^x}{e^x + 1}$

70. $y = 2x - \sin 2x$

Show that the functions in Exercises 21–28 have exactly one zero in the given interval.

28. $r(\theta) = \tan\theta - \cot\theta - \theta, \quad (0, \pi/2)$

16. For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

30. Suppose that $f(0) = 5$ and that $f'(x) = 2$ for all x. Must $f(x) = 2x + 5$ for all x? Give reasons for your answer.

65. Show that $|\cos x - 1| \le |x|$ for all x-values. (*Hint:* Consider $f(t) = \cos t$ on $[0, x]$.)

76. Use the same-derivative argument to prove the identities

a. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ **b.** $\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$

Recall:

1. $\dfrac{d(\sin^{-1} u)}{dx} = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad |u| < 1$
2. $\dfrac{d(\cos^{-1} u)}{dx} = -\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad |u| < 1$
3. $\dfrac{d(\tan^{-1} u)}{dx} = \dfrac{1}{1+u^2}\dfrac{du}{dx}$
4. $\dfrac{d(\cot^{-1} u)}{dx} = -\dfrac{1}{1+u^2}\dfrac{du}{dx}$
5. $\dfrac{d(\sec^{-1} u)}{dx} = \dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}, \quad |u| > 1$
6. $\dfrac{d(\csc^{-1} u)}{dx} = -\dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}, \quad |u| > 1$