Problem Solving_Template

Sunday, November 01, 2020 10:27 PM

3.9 Inverse Trigonometric Functions

Ex) $y = \frac{1}{\arcsin x}$, $y' = ?$

Ex) $y = x \cdot \arctan\sqrt{x}$

39. $y = \tan^{-1}\sqrt{x^2 - 1} + \csc^{-1} x, \quad x > 1$

36. $y = \cos^{-1}(e^{-t})$

31. $y = \cot^{-1}\sqrt{t}$

32. $y = \cot^{-1}\sqrt{t-1}$

33. $y = \ln(\tan^{-1} x)$

34. $y = \tan^{-1}(\ln x)$

35. $y = \tan^{-1}(e^{x^2})$

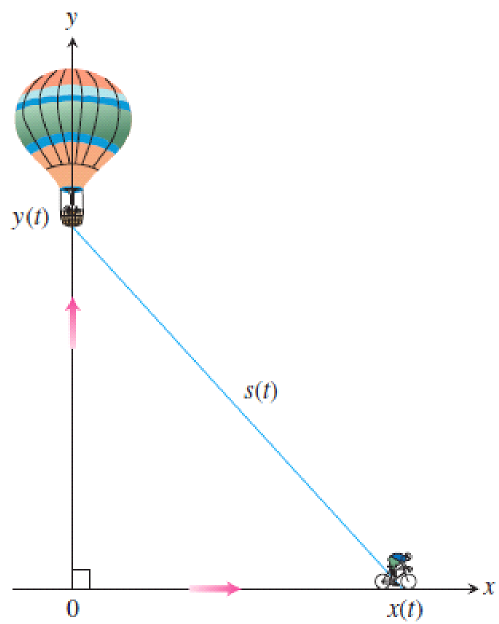
36. $y = \cot^{-1}\left(\frac{1}{x^2}\right) + \cos^{-1} 2x$

37. $y = \sec^{-1}(e^{-x})$

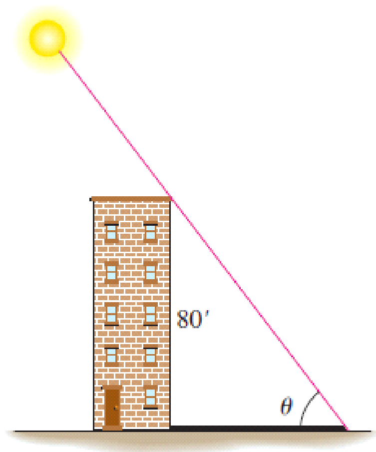
38. $y = e^{\tan^{-1}\sqrt{x^2+1}}$

3.10 Related Rates

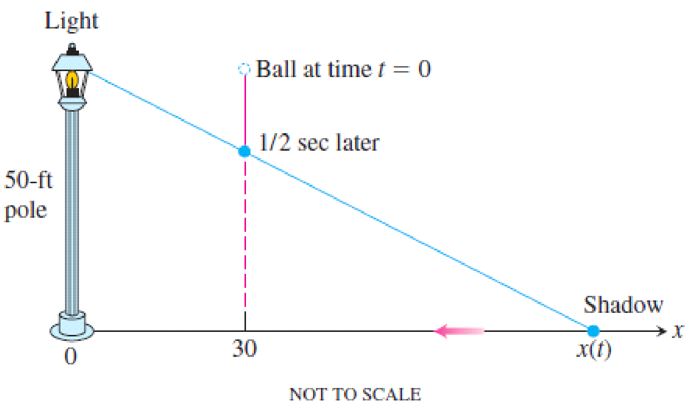
33. A balloon and a bicycle A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance $s(t)$ between the bicycle and balloon increasing 3 sec later?



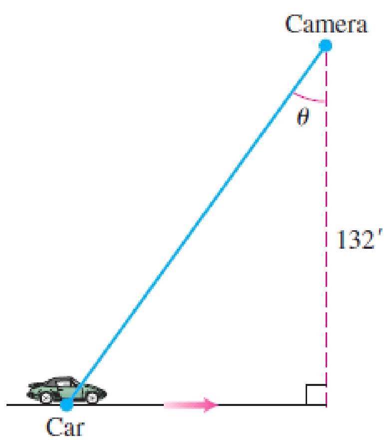
40. A building's shadow On a morning of a day when the sun will pass directly overhead, the shadow of an 80-ft building on level ground is 60 ft long. At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of $0.27°/\text{min}$. At what rate is the shadow decreasing? (Remember to use radians. Express your answer in inches per minute, to the nearest tenth.)



39. A moving shadow A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away from the light. (See accompanying figure.) How fast is the shadow of the ball moving along the ground 1/2 sec later? (Assume the ball falls a distance $s = 16t^2$ ft in t sec.)



38. Videotaping a moving car You are videotaping a race from a stand 132 ft from the track, following a car that is moving at 180 mi/h (264 ft/sec), as shown in the accompanying figure. How fast will your camera angle θ be changing when the car is right in front of you? A half second later?



3.11 Linearization and Differentials

Ex: Find approximately $(0,9)^{9,9}$

Ex: find approximately $\sqrt[3]{8,2}$

Linearization for Approximation

In Exercises 7–14, find a linearization at a suitably chosen integer near x_0 at which the given function and its derivative are easy to evaluate.

11. $f(x) = \sqrt[3]{x}, \quad x_0 = 8.5$

12. $f(x) = \dfrac{x}{x+1}, \quad x_0 = 1.3$

13. $f(x) = e^{-x}, \quad x_0 = -0.1$

15. Show that the linearization of $f(x) = (1 + x)^k$ at $x = 0$ is $L(x) = 1 + kx$.

17. Faster than a calculator Use the approximation $(1 + x)^k \approx 1 + kx$ to estimate the following.

a. $(1.0002)^{50}$ **b.** $\sqrt[3]{1.009}$

Differentials

DEFINITION Let $y = f(x)$ be a differentiable function. The **differential** dx is an independent variable. The **differential** dy is

$$dy = f'(x)\,dx.$$

Derivatives in Differential Form

In Exercises 19–38, find dy.

34. $y = \ln\left(\frac{x+1}{\sqrt{x-1}}\right)$

31. $y = e^{\sqrt{x}}$

38. $y = e^{\tan^{-1}\sqrt{x^2+1}}$

32. $y = xe^{-x}$

Estimating with Differentials

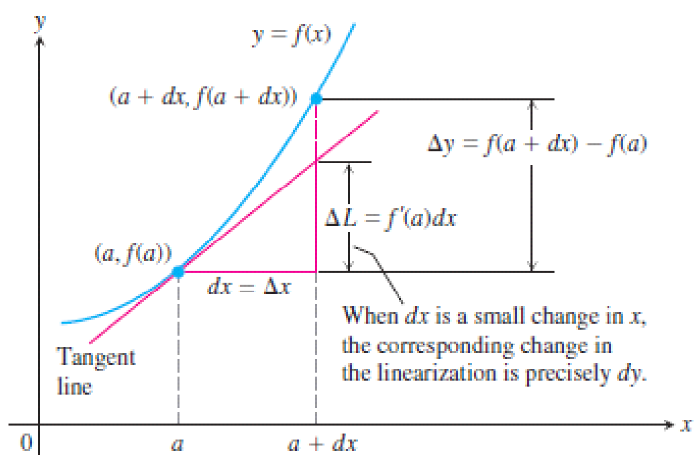


FIGURE 3.54 Geometrically, the differential dy is the change ΔL in the linearization of f when $x = a$ changes by an amount $dx = \Delta x$.

Suppose we know the value of a differentiable function $f(x)$ at a point a and want to estimate how much this value will change if we move to a nearby point $a + dx$. If $dx = \Delta x$ is small, then we can see from Figure 3.54 that Δy is approximately equal to the differential dy. Since

$$f(a + dx) = f(a) + \Delta y, \qquad \Delta x = dx$$

$$f(a + dx) \approx f(a) + dy$$

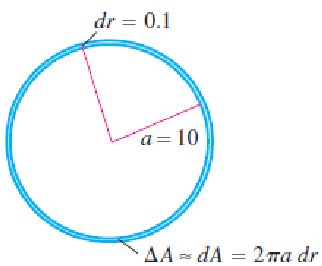


FIGURE 3.55 When dr is small compared with a, the differential dA gives the estimate $A(a + dr) = \pi a^2 + dA$ (Example 6).

EXAMPLE 6 The radius r of a circle increases from $a = 10$ m to 10.1 m (Figure 3.55). Use dA to estimate the increase in the circle's area A. Estimate the area of the enlarged circle and compare your estimate to the true area found by direct calculation.

Solution Since $A = \pi r^2$, the estimated increase is

$$dA = A'(a)\,dr = 2\pi a\,dr = 2\pi(10)(0.1) = 2\pi \text{ m}^2.$$

Thus, since $A(r + \Delta r) \approx A(r) + dA$, we have

$$\begin{aligned} A(10 + 0.1) &\approx A(10) + 2\pi \\ &= \pi(10)^2 + 2\pi = 102\pi. \end{aligned}$$

The area of a circle of radius 10.1 m is approximately 102π m^2.

The true area is

$$\begin{aligned} A(10.1) &= \pi(10.1)^2 \\ &= 102.01\pi \text{ m}^2. \end{aligned}$$

The error in our estimate is 0.01π m^2, which is the difference $\Delta A - dA$.

52. The diameter of a tree was 10 in. During the following year, the circumference increased 2 in. About how much did the tree's diameter increase? The tree's cross-section area?

55. Tolerance The radius r of a circle is measured with an error of at most 2%. What is the maximum corresponding percentage error in computing the circle's

a. circumference? **b.** area?