Problem Solving_Template

27 Ekim 2020 Salı 00:00

Find the first derivative of the following functions:

① $g(t) = \left(\dfrac{1 + \sin 3t}{3 - 2t}\right)^{-1}$

② $y = \sqrt{3t + \sqrt{2 + \sqrt{1 - t}}}$

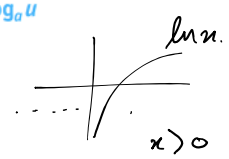
③ $y = \theta(\sin(\ln\theta) + \cos(\ln\theta))$

Derivative of the Natural Logarithm Function

$$\frac{d}{dx}\ln|x| = \frac{1}{x}, \quad x \neq 0$$

The Derivatives of a^u and $\log_a u$

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}.$$



$(3^x)' = 3^x \cdot \ln 3$

$(3^{5x})' = 3^{5x} \cdot \ln 3 \cdot 5$

$(3^{x^2})' = 3^{x^2} \cdot \ln 3 \cdot 2x$

For $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\frac{du}{dx}. \quad (7)$$

$\log_a b = \frac{\ln b}{\ln a}$ $\longrightarrow$ $(\log_2 x)' = \left(\frac{\ln x}{\ln 2}\right)' = \frac{1}{\ln 2}\cdot(\ln x)' = \frac{1}{\ln 2}\cdot\frac{1}{x}$

$\hookrightarrow (\log_x 2)' = \left(\frac{\ln 2}{\ln x}\right)' = \ln 2\cdot\left(\frac{1}{\ln x}\right)' = \ln 2\cdot\frac{(-1)}{(\ln x)^2}\cdot\frac{1}{x}$

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③ Examp: Find $y' = \frac{dy}{dx}$ for the following functions.

① $y = \ln x^3$

② $y = (\ln x)^3$

③ $y = \ln(\ln(\ln x))$

④ $y = \ln\dfrac{1}{x\sqrt{x+1}}$

$\log_a u = \frac{\ln u}{\ln a}$

Ex: $\left(\log_{x^2} x^3 + 5\right)' =$

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⑥ $y = t\log_3\left(e^{(\sin t)(\ln 3)}\right)$

★ $y = \log_5 \sqrt{\left(\frac{7x}{3x + 2}\right)^{\ln 5}}$

Logarithmic Differentiation

Differentiate $f(x) = x^x, x > 0$.

8 $y = (\ln x)^{\ln x}$

★ $x^y = y^x \quad \Rightarrow \frac{dy}{dx} = ?$

★ $y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$ (Logarithmic diff)

$$\ln(x/y) = 1 \quad \Rightarrow \quad \frac{dy}{dx} = ?$$

Find $\left.\frac{dy}{dx}\right|_{x=\frac{1}{6}}$ if $y = \sin(3\pi \cdot \sin(2\pi \cdot \sin(\pi x)))$

Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	−4	−1/3	−8/3

Find the derivatives with respect to x of the following combinations at the given value of x.

a. $5f(x) - g(x), \quad x = 1$

b. $f(x)g^3(x), \quad x = 0$

c. $\dfrac{f(x)}{g(x) + 1}, \quad x = 1$

d. $f(g(x)), \quad x = 0$

e. $g(f(x)), \quad x = 0$

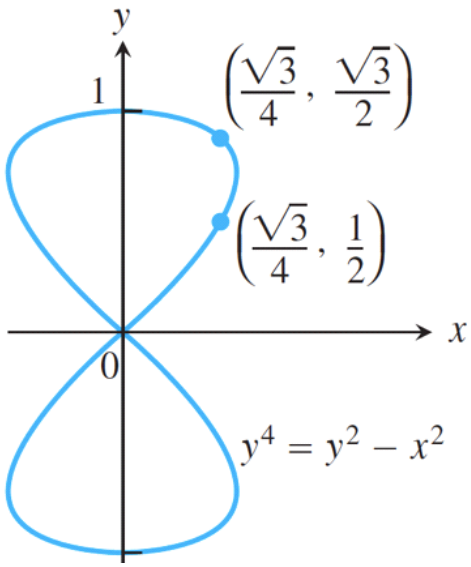
f. $(x^{11} + f(x))^{-2}, \quad x = 1$

g. $f(x + g(x)), \quad x = 0$

If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point $(2, 2)$.

Tangent parabola The parabola $y = x^2 + C$ is to be tangent to the line $y = x$. Find C.

The eight curve Find the slopes of the curve $y^4 = y^2 - x^2$ at the two points shown here.



Is there anything special about the tangents to the curves $y^2 = x^3$ and $2x^2 + 3y^2 = 5$ at the points $(1, \pm 1)$? Give reasons for your answer.