Problem Solving_Template

3.1 Tangents and the Derivative at a Point

In Exercises 19–22, find the slope of the curve at the point indicated.

19. $y = 5x^2, \quad x = -1$

20. $y = 1 - x^2, \quad x = 2$

21. $y = \dfrac{1}{x - 1}, \quad x = 3$

22. $y = \dfrac{x - 1}{x + 1}, \quad x = 0$

In Exercises 11–18, find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

11. $f(x) = x^2 + 1, \quad (2, 5)$

12. $f(x) = x - 2x^2, \quad (1, -1)$

13. $g(x) = \dfrac{x}{x - 2}, \quad (3, 3)$

14. $g(x) = \dfrac{8}{x^2}, \quad (2, 2)$

15. $h(t) = t^3, \quad (2, 8)$

16. $h(t) = t^3 + 3t, \quad (1, 4)$

17. $f(x) = \sqrt{x}, \quad (4, 2)$

18. $f(x) = \sqrt{x + 1}, \quad (8, 3)$

Tangent Lines with Specified Slopes

At what points do the graphs of the functions in Exercises 23 and 24 have horizontal tangents?

23. $f(x) = x^2 + 4x - 1$

24. $g(x) = x^3 - 3x$

25. Find equations of all lines having slope -1 that are tangent to the curve $y = 1/(x - 1)$.

26. Find an equation of the straight line having slope $1/4$ that is tangent to the curve $y = \sqrt{x}$.

Testing for Tangents

33. Does the graph of

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

have a tangent at the origin? Give reasons for your answer.

34. Does the graph of

$$g(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

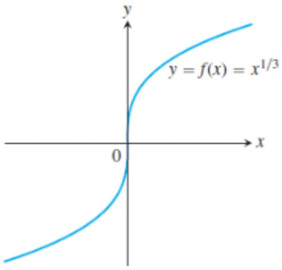
have a tangent at the origin? Give reasons for your answer.

b. Show that

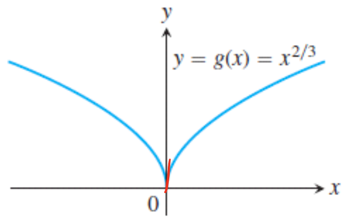
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is differentiable at $x = 0$ and find $f'(0)$.

Remark



VERTICAL TANGENT AT ORIGIN



NO VERTICAL TANGENT AT ORIGIN

36. Does the graph of

$$U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

have a vertical tangent at the point (0, 1)? Give reasons for your answer.

3.2 The Derivative as a Function

In Exercises 7–12, find the indicated derivatives.

★ **9.** $\dfrac{ds}{dt}$ if $s = \dfrac{t}{2t + 1}$ **10.** $\dfrac{dv}{dt}$ if $v = t - \dfrac{1}{t}$

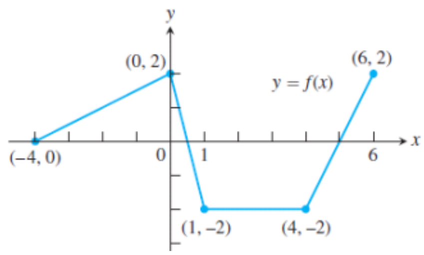
★ **11.** $\dfrac{dp}{dq}$ if $p = \dfrac{1}{\sqrt{q + 1}}$ **12.** $\dfrac{dz}{dw}$ if $z = \dfrac{1}{\sqrt{3w - 2}}$

In Exercises 17–18, differentiate the functions. Then find an equation of the tangent line at the indicated point on the graph of the function.

17. $y = f(x) = \dfrac{8}{\sqrt{x-2}}, \quad (x, y) = (6, 4)$

18. $w = g(z) = 1 + \sqrt{4 - z}, \quad (z, w) = (3, 2)$

31. a. The graph in the accompanying figure is made of line segments joined end to end. At which points of the interval $[-4, 6]$ is f' not defined? Give reasons for your answer.



In Exercises 41 and 42, determine if the piecewise defined function is differentiable at the origin.

41. $f(x) = \begin{cases} 2x - 1, & x \ge 0 \\ x^2 + 2x + 7, & x < 0 \end{cases}$

Differentiability and Continuity on an Interval

Each figure in Exercises 43–48 shows the graph of a function over a closed interval D. At what domain points does the function appear to be

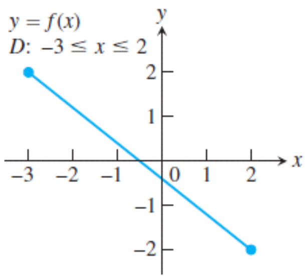
a. differentiable?

b. continuous but not differentiable?

c. neither continuous nor differentiable?

Give reasons for your answers.

43.



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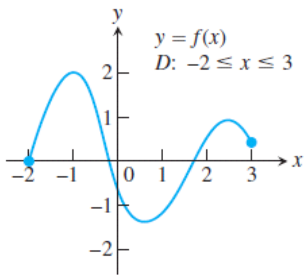

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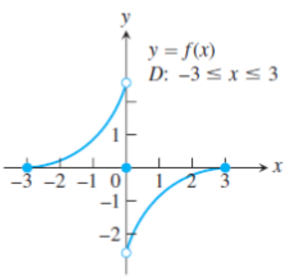

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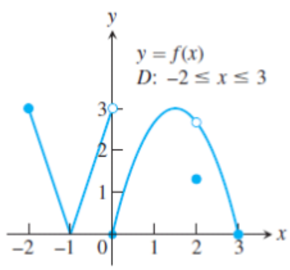

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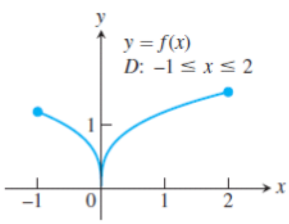

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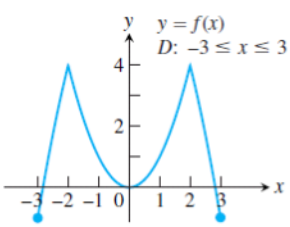

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53. Tangent to a parabola Does the parabola $y = 2x^2 - 13x + 5$ have a tangent whose slope is -1? If so, find an equation for the line and the point of tangency. If not, why not?

3.3 Differentiation Rules

Find the derivatives of the functions

29. $y = 2e^{-x} + e^{3x}$

30. $y = \dfrac{x^2 + 3e^x}{2e^x - x}$

31. $y = x^3e^x$

32. $w = re^{-r}$

33. $y = x^{9/4} + e^{-2x}$

34. $y = x^{-3/5} + \pi^{3/2}$

35. $s = 2t^{3/2} + 3e^2$

36. $w = \dfrac{1}{z^{1.4}} + \dfrac{\pi}{\sqrt{z}}$

37. $y = \sqrt[7]{x^2} - x^e$

38. $y = \sqrt[3]{x^{9.6}} + 2e^{1.3}$

39. $r = \dfrac{e^s}{s}$

40. $r = e^{\theta}\left(\dfrac{1}{\theta^2} + \theta^{-\pi/2}\right)$

23. $f(s) = \dfrac{\sqrt{s} - 1}{\sqrt{s} + 1}$

51. $w = 3z^2e^{2z}$

3.5 Derivatives of Trigonometric Functions

Derivatives

In Exercises 1–18, find dy/dx.

1. $y = -10x + 3\cos x$
2. $y = \dfrac{3}{x} + 5\sin x$
3. $y = x^2\cos x$
4. $y = \sqrt{x}\sec x + 3$
5. $y = \csc x - 4\sqrt{x} + 7$
6. $y = x^2\cot x - \dfrac{1}{x^2}$
7. $f(x) = \sin x \tan x$
8. $g(x) = \csc x \cot x$
9. $y = (\sec x + \tan x)(\sec x - \tan x)$
10. $y = (\sin x + \cos x)\sec x$