Problem Solving_Template

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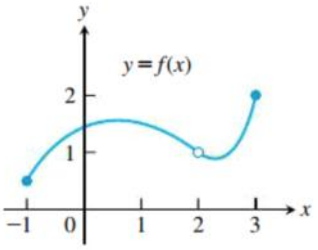
Continuity from Graphs

In Exercises 1–4, say whether the function graphed is continuous on $[-1, 3]$. If not, where does it fail to be continuous and why?

1.



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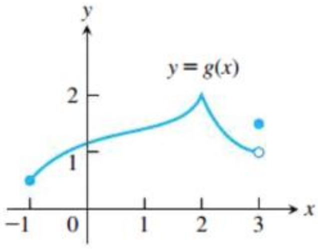

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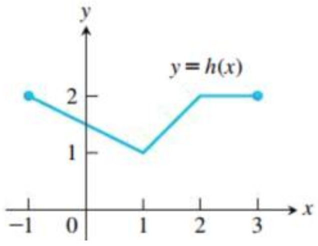

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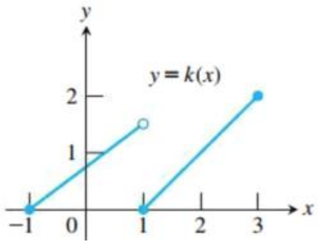
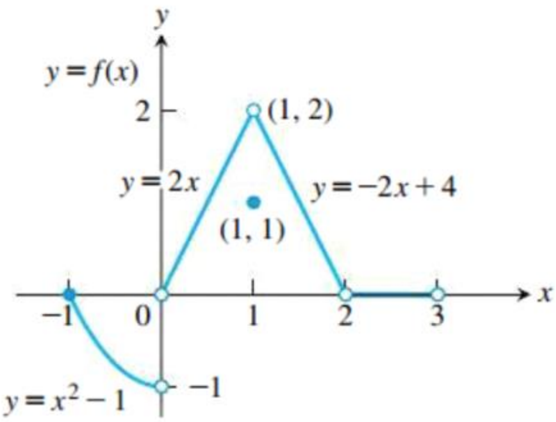


Exercises 5–10 refer to the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$

graphed in the accompanying figure.



The graph for Exercises 5–10.

5. a. Does $f(-1)$ exist?

b. Does $\lim_{x \to -1^+} f(x)$ exist?

c. Does $\lim_{x \to -1^+} f(x) = f(-1)$?

d. Is f continuous at $x = -1$?

6. a. Does $f(1)$ exist?

b. Does $\lim_{x \to 1} f(x)$ exist?

c. Does $\lim_{x \to 1} f(x) = f(1)$?

d. Is f continuous at $x = 1$?

7. a. Is f defined at $x = 2$? (Look at the definition of f.)

b. Is f continuous at $x = 2$?

8. At what values of x is f continuous?

9. What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?

10. To what new value should $f(1)$ be changed to remove the discontinuity?

Limits Involving Trigonometric Functions

Find the limits in Exercises 31–38. Are the functions continuous at the point being approached?

31. $\lim_{x\to\pi} \sin(x - \sin x)$

32. $\lim_{t\to 0} \sin\left(\frac{\pi}{2}\cos(\tan t)\right)$

33. $\lim_{y\to 1} \sec(y\sec^2 y - \tan^2 y - 1)$

34. $\lim_{x\to 0} \tan\left(\frac{\pi}{4}\cos(\sin x^{1/3})\right)$

35. $\lim_{t\to 0} \cos\left(\frac{\pi}{\sqrt{19 - 3\sec 2t}}\right)$

36. $\lim_{x\to\pi/6} \sqrt{\csc^2 x + 5\sqrt{3}\tan x}$

37. $\lim_{x\to 0^+} \sin\left(\frac{\pi}{2}e^{\sqrt{x}}\right)$

38. $\lim_{x\to 1} \cos^{-1}(\ln\sqrt{x})$

Past Exam Question

6. (16 points) Let

$$f(x) = \begin{cases} x - c & \text{if } x < 2 \\ b & \text{if } x = 2 \\ -cx^2 + 8 & \text{if } x > 2 \end{cases}$$

(a) (6 points) Find the values of b and c that make the function $f(x)$ continuous at $x = 2$.

Use the Intermediate Value Theorem in Exercises 71–78 to prove that each equation has a solution.

78. $2\sin x = x$ (three roots).

2.6 Limits involving infiny, Asymtotes of graphs

Limits of Rational Functions

In Exercises 13–22, find the limit of each rational function **(a)** as $x \to \infty$ and **(b)** as $x \to -\infty$.

17. $h(x) = \dfrac{7x^3}{x^3 - 3x^2 + 6x}$

18. $h(x) = \dfrac{9x^4 + x}{2x^4 + 5x^2 - x + 6}$

19. $g(x) = \dfrac{10x^5 + x^4 + 31}{x^6}$

20. $g(x) = \dfrac{x^3 + 7x^2 - 2}{x^2 - x + 1}$

Limits as $x \to \infty$ or $x \to -\infty$

The process by which we determine limits of rational functions applies equally well to ratios containing noninteger or negative powers of x: Divide numerator and denominator by the highest power of x in the denominator and proceed from there. Find the limits in Exercises 23–36.

→ **33.** $\lim_{x \to \infty} \dfrac{\sqrt{x^2 + 1}}{x + 1}$

→ **34.** $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 + 1}}{x + 1}$

Finding Limits of Differences When $x \to \pm\infty$

Find the limits in Exercises 86–92. (*Hint*: Try multiplying and dividin; by the conjugate.)

86. $\lim_{x \to \infty} (\sqrt{x + 9} - \sqrt{x + 4})$

87. $\lim_{x \to \infty} (\sqrt{x^2 + 25} - \sqrt{x^2 - 1})$

88. $\lim_{x \to -\infty} (\sqrt{x^2 + 3} + x)$

89. $\lim_{x \to -\infty} (2x + \sqrt{4x^2 + 3x - 2})$

90. $\lim_{x \to \infty} (\sqrt{9x^2 - x} - 3x)$

91. $\lim_{x \to \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x})$

92. $\lim_{x \to \infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x})$

RECALL: Infinite Limits

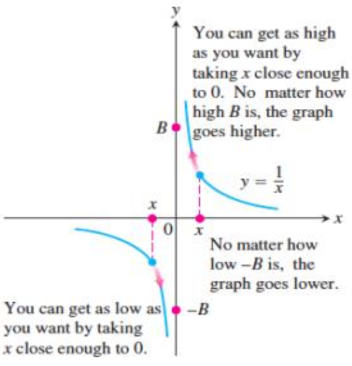


FIGURE 2.59 One-sided infinite limits: $\lim_{x\to 0^+} \frac{1}{x} = \infty$ and $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

Let us look again at the function $f(x) = 1/x$. As $x \to 0^+$, the values of f grow without bound, eventually reaching and surpassing every positive real number. That is, given any positive real number B, however large, the values of f become larger still (Figure 2.59). Thus, f has no limit as $x \to 0^+$. It is nevertheless convenient to describe the behavior of f by saying that $f(x)$ approaches ∞ as $x \to 0^+$. We write

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1}{x} = \infty.$$

In writing this equation, we are *not* saying that the limit exists. Nor are we saying that there is a real number ∞, for there is no such number. Rather, we are saying that $\lim_{x\to 0^+} (1/x)$ *does not exist because $1/x$ becomes arbitrarily large and positive as $x \to 0^+$.*

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53. $\lim \dfrac{1}{x^2 - 4}$ as

a. $x \to 2^+$

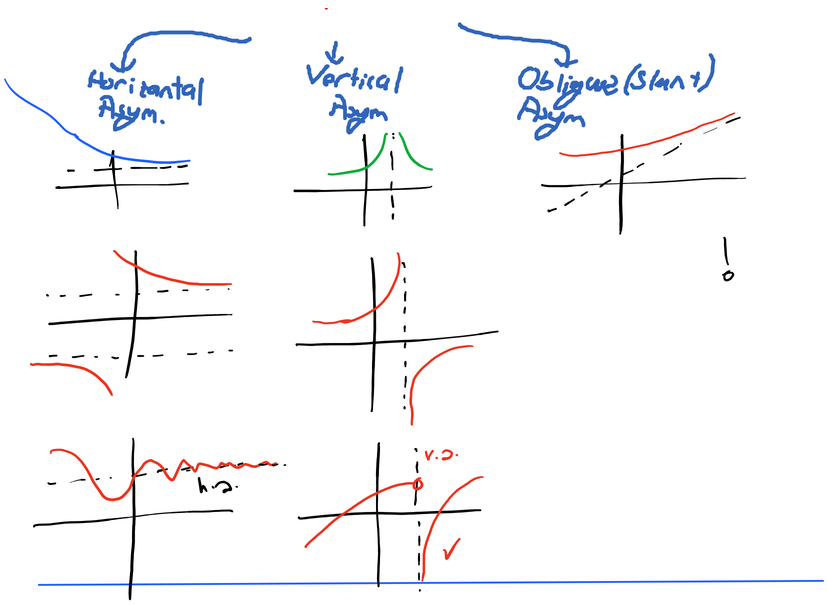
b. $x \to 2^-$

c. $x \to -2^+$

d. $x \to -2^-$

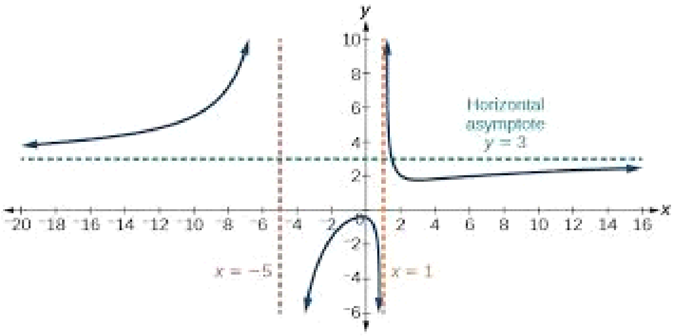
Asymtotes

Horizantal Vertical Obligue (Slant)



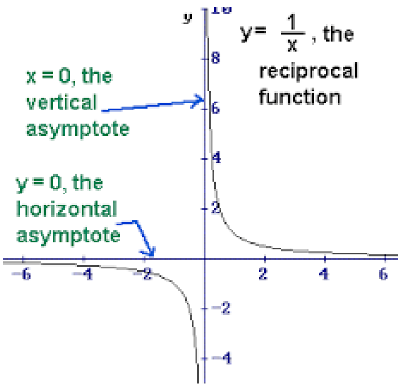
DEFINITION A line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b.$$



DEFINITION A line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if either

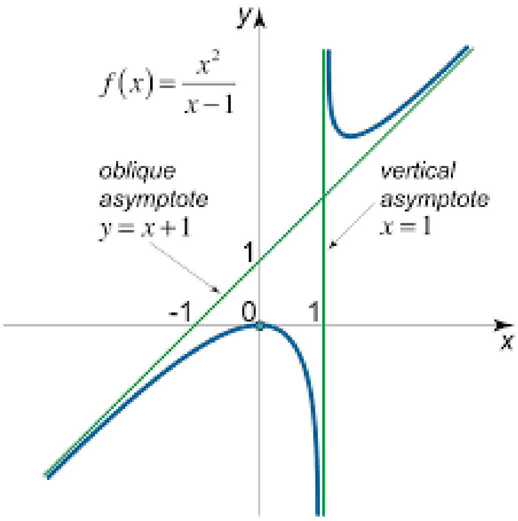
$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty.$$

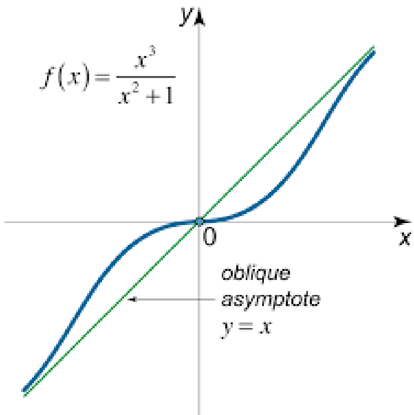


The straight line $y = ax + b$ (where $a \neq 0$) is an **oblique asymptote** of the graph of $y = f(x)$ if

$$\text{either} \quad \lim_{x \to -\infty} \big(f(x) - (ax + b)\big) = 0 \quad \text{or} \quad \lim_{x \to \infty} \big(f(x) - (ax + b)\big) = 0,$$

or both.





EXAMPLE 15 Find the horizontal and vertical asymptotes of the curve

$$y = \frac{x+3}{x+2}.$$

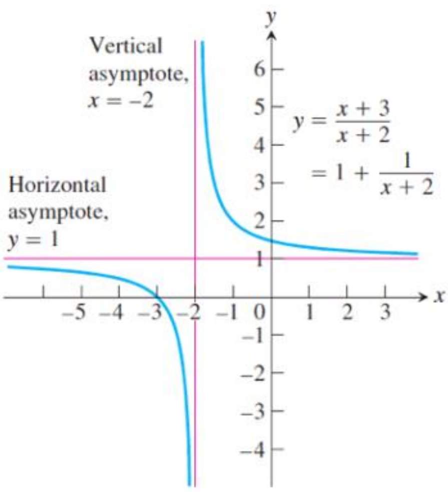
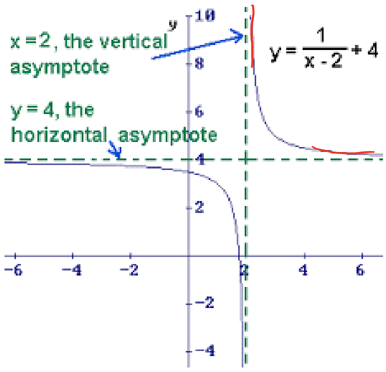


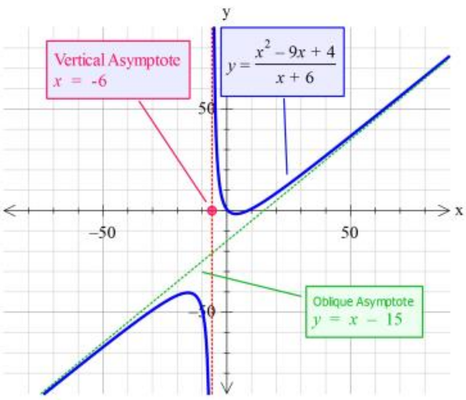
FIGURE 2.65 The lines $y = 1$ and $x = -2$ are asymptotes of the curve in Example 15.

Ex)



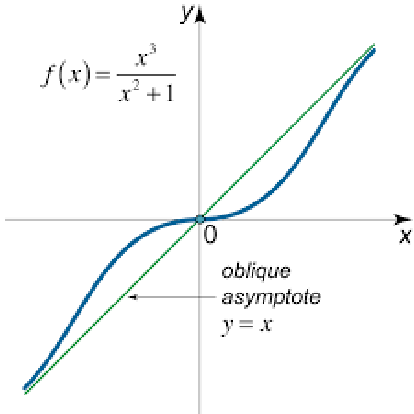
x = 2, the vertical asymptote
y = 4, the horizontal asymptote
$y = \frac{1}{x-2} + 4$

Ex)

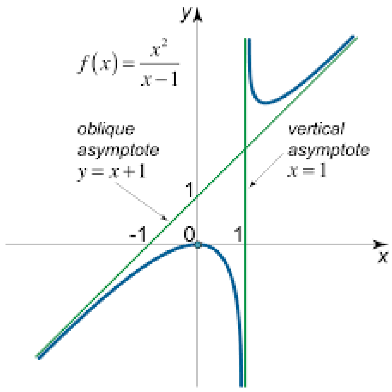


Vertical Asymptote
x = -6
$y = \frac{x^2 - 9x + 4}{x + 6}$
Oblique Asymptote
y = x − 15

Ex)



Ex)



Ex)

Find the oblique asymptote of graphs of the rational functions

99. $y=\dfrac{x^2}{x-1}$

100. $y=\dfrac{x^2+1}{x-1}$

101. $y=\dfrac{x^2-4}{x-1}$

102. $y=\dfrac{x^2-1}{2x+4}$

103. $y=\dfrac{x^2-1}{x}$

104. $y=\dfrac{x^3+1}{x^2}$

Past Exam Question

4. (15 points) Find all asymptotes of the graph $y = \dfrac{(x+3)(x+2)(x+1)}{x^2+x-2}$, if exists. Classify them as vertical, horizontal and oblique (i.e. slant).