Problem Solving_sec03

Saturday, December 19, 2020 7:13 PM

6.1 Volumes Using Cross-Sections

EXAMPLE 2 A curved wedge is cut from a circular cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.

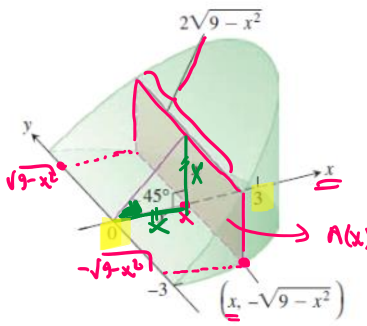

FIGURE 6.6 The wedge of Example 2, sliced perpendicular to the x-axis. The cross-sections are rectangles.

base height

$A(x) = \underbrace{2\sqrt{9-x^2}}_{\text{base}} \cdot \underbrace{x}_{\text{height}}$

$$V = \int_0^3 A(x)\,dx$$

$$V = \int_0^3 2\sqrt{9-x^2}\cdot x\,dx$$

$u = 9 - x^2$

$du = -2x\,dx$, $\quad x\,dx = -\frac{du}{2}$

$$= -\int \sqrt{u}\,du$$

⋮

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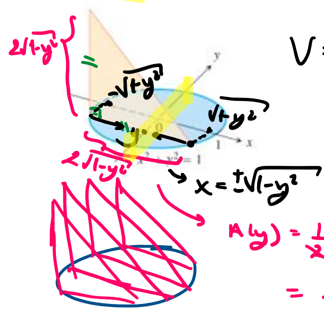
10. The base of the solid is the disk $x^2 + y^2 \le 1$. The cross-sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk.



$$V = \int A(y)\,dy = \int_{-1}^{+1} 2(1-y^2)\,dy$$

$x = \pm\sqrt{1-y^2}$

$A(y) = \frac{1}{2}\cdot 2\sqrt{1-y^2}\cdot 2\sqrt{1-y^2}$

$= 2(1-y^2)$

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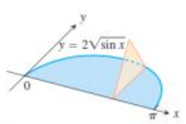
5. The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross-sections perpendicular to the x-axis are

a. equilateral triangles with bases running from the x-axis to the curve as shown in the accompanying figure.

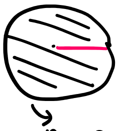


b. squares with bases running from the x-axis to the curve.

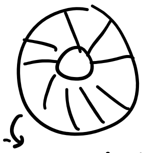
Recall

Volume of solid objects obtained by rotating region around some axes

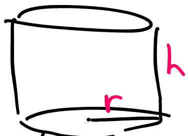
Disk Method ⟷ Washer Method | Cylindricall Shell Method.



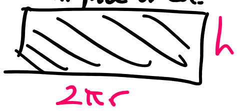
Area of disk



Area of ring



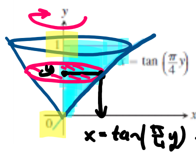
Surface area.



Volumes by the Disk Method

In Exercises 15–18, find the volume of the solid generated by revolving the shaded region about the given axis.

17. About the y-axis



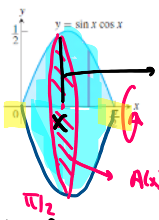
$x = \tan\left(\frac{\pi}{4}y\right) = R(y)$

$$V = \int_0^1 A(y)\,dy$$

$$= \int_0^1 \pi\, R(y)^2\,dy$$

$$= \int_0^1 \pi \tan\left(\frac{\pi}{4}y\right)^2 dy$$

18. About the x-axis



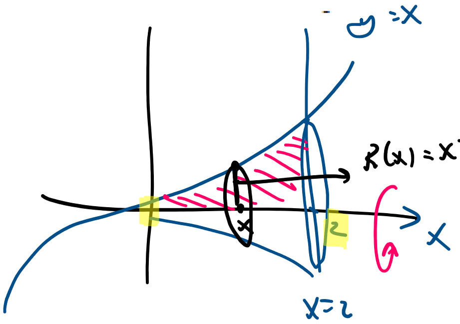
$R(x) = \sin x \cos x$

$A(x) = \pi R^2(x) = \pi(\sin x \cos x)^2$

$$V = \int_0^{\pi/2} A(x)\,dx = \int_0^{\pi/2} \pi (\sin x \cos x)^2\,dx$$

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 19–28 about the x-axis.

20. $y = x^3, \quad y = 0, \quad x = 2$



$R(x) = x^3$

$$V = \int_0^2 A(x)\,dx$$

$$= \int_0^2 \pi\, R(x)^2\,dx$$

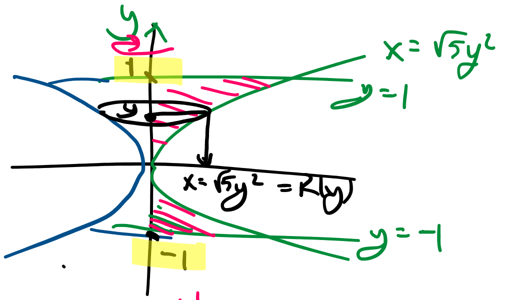
$$= \int_0^2 \pi (x^3)^2\,dx$$

25. $y = e^{-x}, \quad y = 0, \quad x = 0, \quad x = 1$

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 31–36 about the y-axis.

31. The region enclosed by $x = \sqrt{5}\, y^2, \quad x = 0, \quad y = -1, \quad y = 1$

32. The region enclosed by $x = y^{3/2}, \quad x = 0, \quad y = 2$



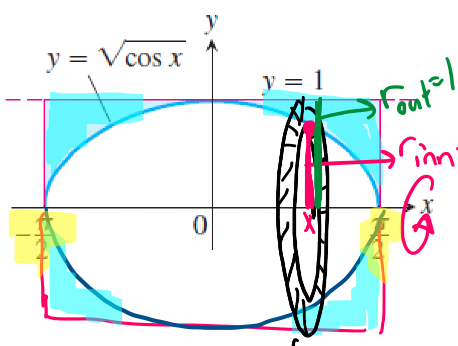
$$V = \int_{-1}^{+1} A(y)\, dy$$

$$= \int_{-1}^{+1} \pi \left(\sqrt{5}y^2\right)^2 dy$$

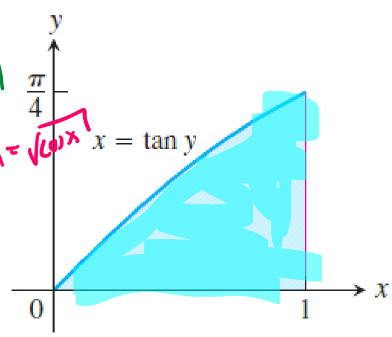
Volumes by the Washer Method

Find the volumes of the solids generated by revolving the shaded regions in Exercises 37 and 38 about the indicated axes.

37. The x-axis



38. The y-axis

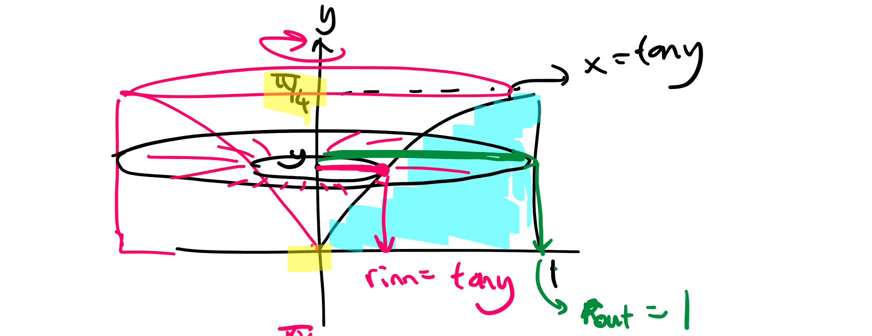


$$A(x) = \pi r_{out}^2 - \pi r_{inn}^2$$
$$= \pi\, 1^2 - \pi\left(\sqrt{\cos x}\right)^2$$

$$V = \int_{-\pi/2}^{\pi/2} A(x)\, dx$$

$$= \int_{-\pi/2}^{\pi/2} \pi \left(r_{out}^2 - r_{inn}^2 \right) dx$$

$$= \int_{-\pi/2}^{\pi/2} \pi \left(1 - (\sqrt{\cos x})^2\right) dx$$



$$V = \int_0^{\pi/4} A(y)\, dy$$

$$= \int_0^{\pi/4} \pi \left(r_{out}^2 - r_{inn}^2\right) dy$$

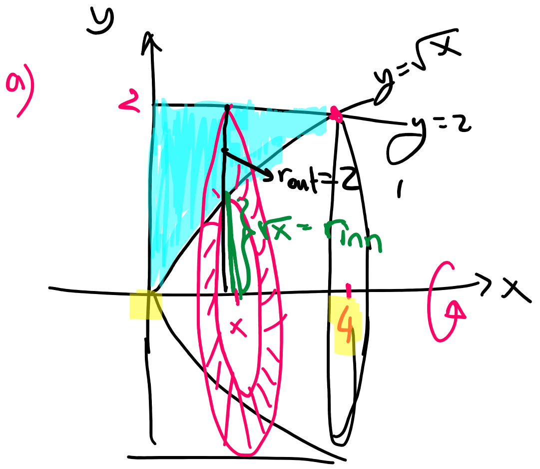
$$= \int_0^{\pi/4} \pi \left(1^2 - \tan^2 y\right) dy$$

51. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about

a. the x-axis. **b.** the y-axis.

c. the line $y = 2$. **d.** the line $x = 4$.

a)

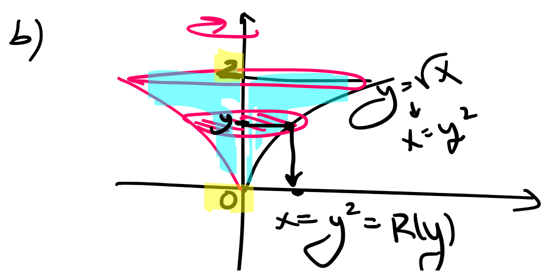


$$V = \int_0^4 A(x)\, dx$$

$$= \int_0^4 \pi \left(r_{out}^2 - r_{inn}^2\right) dx$$

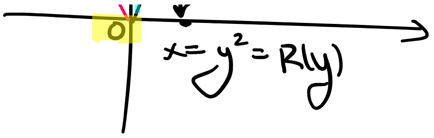
$$= \int_0^4 \pi \left(2^2 - (\sqrt{x})^2\right) dx$$

b)



$$V = \int_0^2 A(y)\, dy$$

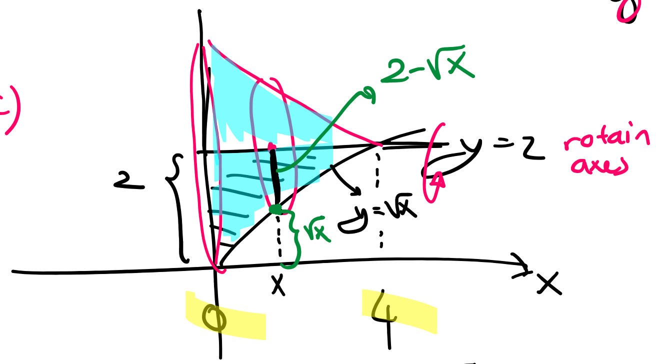
$$= \int_0^2 \pi\, R(y)^2\, dy$$



$$= \int_0 \pi \; R(y) \, dy$$

$$= \int_0^2 \pi \, (y^2)^2 \, dy$$

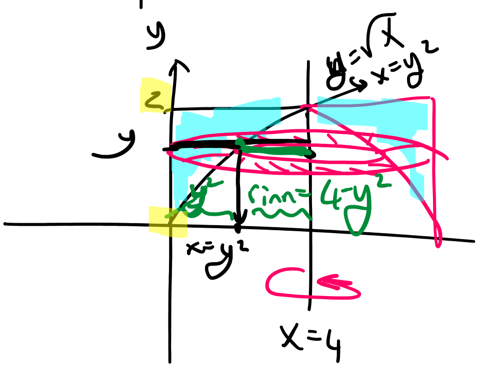
c)



$$V = \int_0^4 \pi \, R(x)^2 \, dx$$

$$= \int_0^4 \pi \, (2-\sqrt{x})^2 \, dx$$

d)



$$V = \int_0^2 A(y) \, dy$$

$$V = \int_0^2 \pi \, (r_{out}^2 - r_{inn}^2) \, dy$$

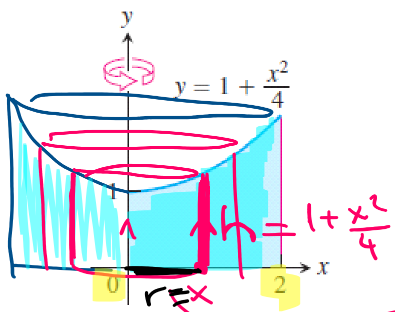
$$= \int_0^2 \pi \, \left(4^2 - (4-y^2)^2\right) dy$$

6.2 Volumes Using Cylindrical Shells

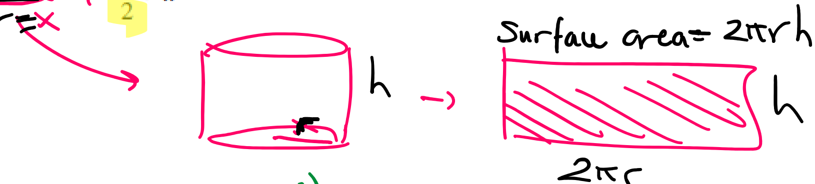
Revolution About the Axes

In Exercises 1–6, use the shell method to find the volumes of the solids generated by revolving the shaded region about the indicated axis.

1.



$$V = \int_0^2 2\pi \underset{= x}{r} \, \underset{= 1+\frac{x^2}{4}}{h} \, dx$$



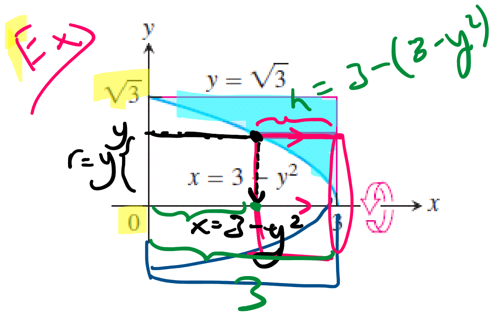
4.

y

$2\pi r$

4.

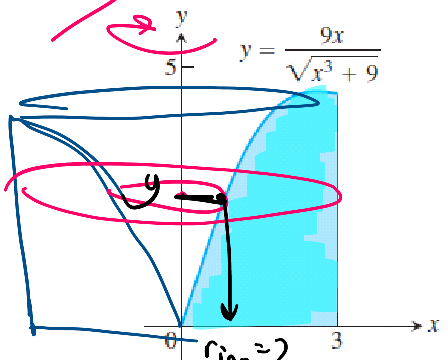
Ex)



$$V = \int_0^{\sqrt{3}} 2\pi \underset{\substack{\shortparallel \\ y}}{r} \; \underset{\substack{\shortparallel \\ (3-(3-y^2))}}{h} \; dy$$

$$= \int_0^{\sqrt{3}} 2\pi \overset{r}{y} \overbrace{(3-(3-y^2))}^{h} \, dy$$

Ex) 6. The y-axis



Washer Method

$V = \int A(y)\,dy$

$r_{inn} = ?$ → we need to solve

$y = \dfrac{9x}{\sqrt{x^3+9}}$ for "x" = ?

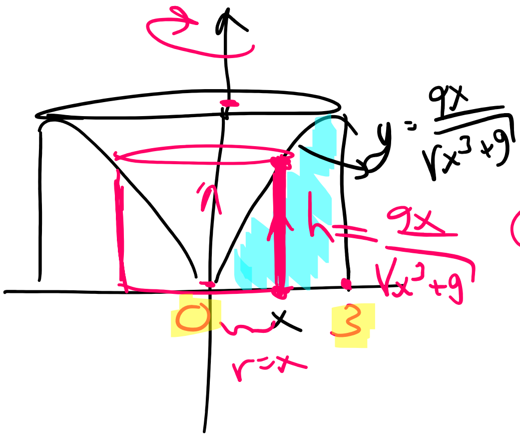
which is not possible!

⇓

We can not use washer Method

⇓

We have to use Cyl. Shell method

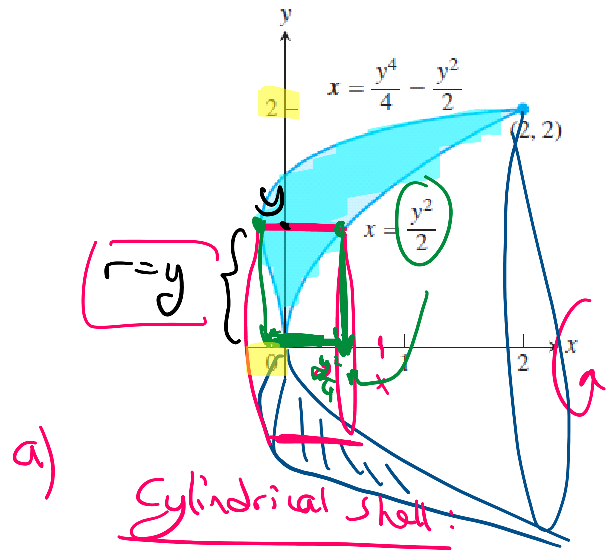


Cylindrical Shell:

$$V = \int_0^3 2\pi \underset{\substack{\shortparallel \\ x}}{r} \; \underset{\substack{\shortparallel \\ \frac{9x}{\sqrt{x^3+9}}}}{h} \; dx$$

28. **a.** The x-axis **b.** The line $y = 2$

c. The line $y = 5$ **d.** The line $y = -5/8$

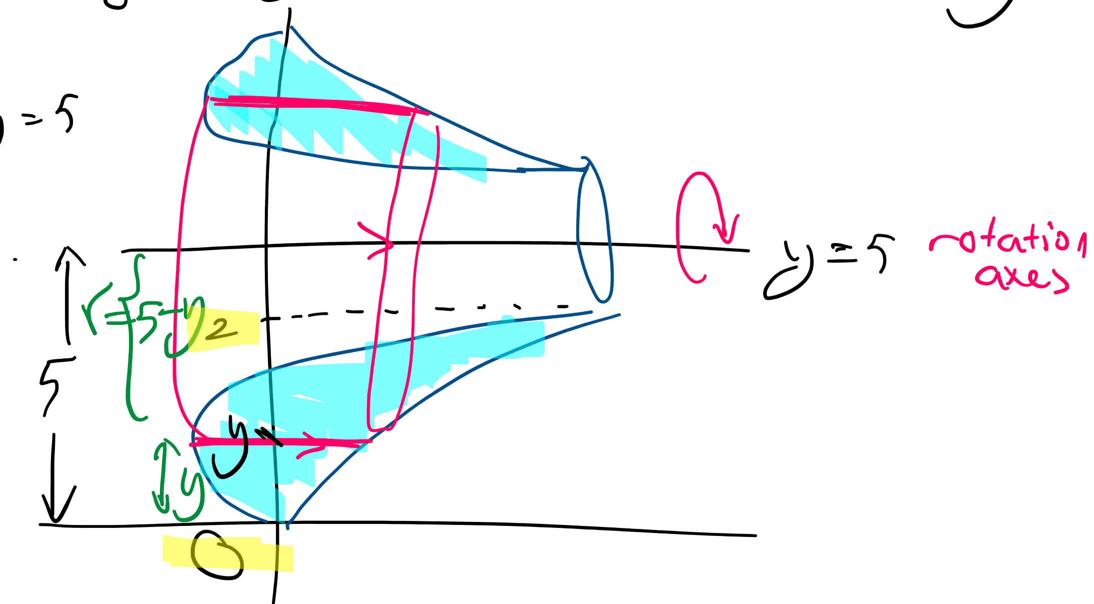


a) Cylindrical shell:

$$V = \int_0^2 2\pi \underset{\substack{\| \\ y}}{r}\, \underset{\substack{\| \\ \left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right]}}{h}\, dy$$

$$= \int_0^2 2\pi y \left(\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right) dy$$

c) $y = 5$



$$V = \int_0^2 2\pi \underset{\substack{\| \\ 5-y}}{r}\, \underset{\substack{\| \\ \left(\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right)}}{h}\, dy$$

0

$\int y\left(\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right)$

Choosing the Washer Method or Shell Method

29. Compute the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about each coordinate axis using

a. the shell method. **b.** the washer method.

33. The region in the first quadrant bounded by the curve $x = y - y^3$ and the y-axis about

a. the x-axis **b.** the line $y = 1$

30. Compute the volume of the solid generated by revolving the triangular region bounded by the lines $2y = x + 4$, $y = x$, and $x = 0$ about

a. the x-axis using the washer method.

b. the y-axis using the shell method.

c. the line $x = 4$ using the shell method.

d. the line $y = 8$ using the washer method.

39. The region shown here is to be revolved about the x-axis to generate a solid. Which of the methods (disk, washer, shell) could you use to find the volume of the solid? How many integrals would be required in each case? Explain.