# Problem Solving (Template)

13 Aralık 2020 Pazar 23:39

## 5.4 The Fundamental Theorem of Calculus

Evaluate the integrals

**23.** $\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$

**24.** $\int_1^{8} \frac{(x^{1/3} + 1)(2 - x^{2/3})}{x^{1/3}}\, dx$

**25.** $\int_{\pi/2}^{\pi} \frac{\sin 2x}{2 \sin x}\, dx$

**26.** $\int_0^{\pi/3} (\cos x + \sec x)^2\, dx$

**27.** $\int_{-4}^{4} |x|\, dx$

**28.** $\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\, dx$

**29.** $\int_0^{\ln 2} e^{3x}\, dx$

**30.** $\int_1^{2} \left(\frac{1}{x} - e^{-x}\right) dx$

**31.** $\int_0^{1/2} \frac{4}{\sqrt{1 - x^2}}\, dx$

**32.** $\int_0^{1/\sqrt{3}} \frac{dx}{1 + 4x^2}$

**33.** $\int_2^{4} x^{\pi - 1}\, dx$

**34.** $\int_{-1}^{0} \pi^{x-1}\, dx$

## Differentiating Integrals

In Exercises 121–128, find $dy/dx$.

**121.** $y = \int_2^x \sqrt{2 + \cos^3 t}\, dt$

**122.** $y = \int_2^{7x^2} \sqrt{2 + \cos^3 t}\, dt$

**123.** $y = \int_x^1 \frac{6}{3 + t^4}\, dt$

**124.** $y = \int_{\sec x}^2 \frac{1}{t^2 + 1}\, dt$

**125.** $y = \int_{\ln x^2}^0 e^{\cos t}\, dt$

**126.** $y = \int_1^{e^{\sqrt{x}}} \ln(t^2 + 1)\, dt$

**127.** $y = \int_0^{\sin^{-1} x} \frac{dt}{\sqrt{1 - 2t^2}}$

**128.** $y = \int_{\tan^{-1} x}^{\pi/4} e^{\sqrt{t}}\, dt$

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Ex: If $F(x) = \int_1^x f(t)\,dt$ and $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$ then find $F''(2)$.

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Ex: (a) Let $f(x)$ and $g(x)$ be differentiable everywhere, $f(0) = f'(0)$ and $g(x) = (1 - x)\int_x^{x^3} g(t)dt + f(x)$. Find $g'(0)$.

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(b) Let $f(x)$ be differentiable everywhere. Write in the most simplified form of the followings.

i. $\frac{d}{dx}\int_0^{2^x} f(t)dt$

ii. $\frac{d}{dt}\int_{2^x}^{0} f(t)dt$

iii. $\int_0^{2^x} \frac{d}{dx} f(t)dt$

iv. $\int_{2^x}^{0} \frac{d}{dt} f(t)dt$

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Ex:

$$\lim_{x\to 0} \frac{\int_0^x \sin(xt^3)\,dt}{x^5}$$

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## 5.5 Indefinite Integrals and the Substitution Method

$$\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2}\, dx$$

$$\int \frac{1}{\theta^2} \sin \frac{1}{\theta} \cos \frac{1}{\theta}\, d\theta$$

$$\int \sqrt{\frac{x^3 - 3}{x^{11}}}\, dx$$

$$\int x(x - 1)^{10}\, dx$$

$$\int (\sin 2\theta)\, e^{\sin^2 \theta}\, d\theta$$

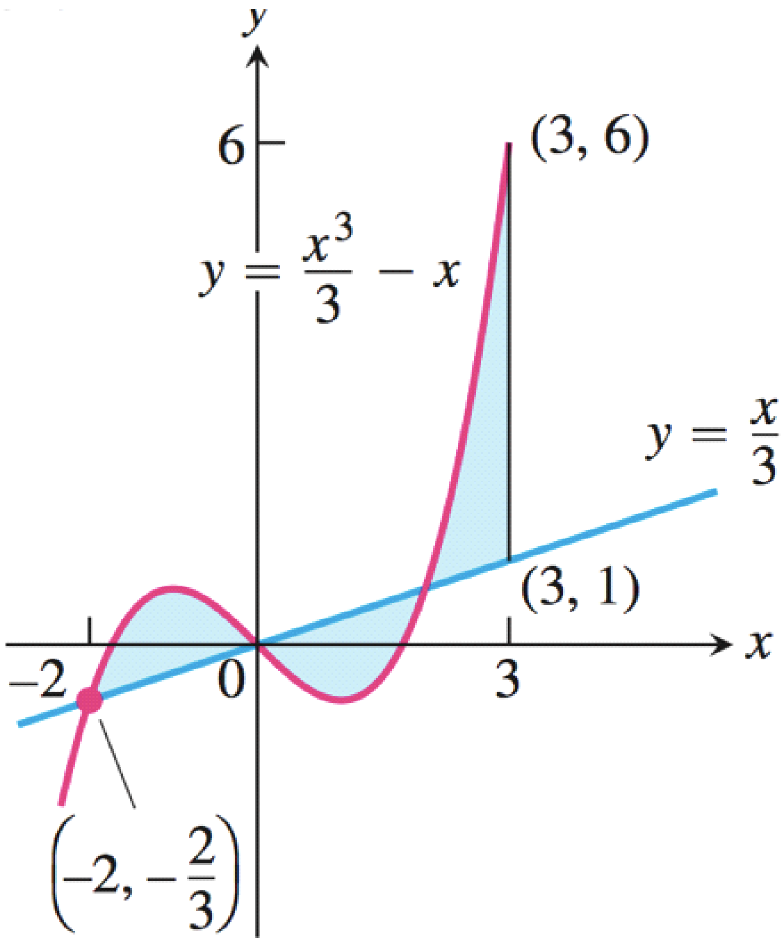
$$\int \frac{dx}{(1+\sqrt{x})^4}$$

## 5.6 Substitution and Area Between Curves

$$\int_{\sqrt{2}}^{2} \frac{\sec^2(\sec^{-1} x)\, dx}{x\sqrt{x^2 - 1}}$$

## Area

Find the total areas of the shaded regions



Find the areas of the regions enclosed by the curves in Exercises 81–84.

**81.** $4x^2 + y = 4$ and $x^4 - y = 1$

**82.** $x^3 - y = 0$ and $3x^2 - y = 4$

**83.** $x + 4y^2 = 4$ and $x + y^4 = 1$, for $x \geq 0$

**84.** $x + y^2 = 3$ and $4x + y^2 = 0$

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Find the area of the "triangular" region in the first quadrant bounded on the left by the $y$-axis and on the right by the curves $y = \sin x$ and $y = \cos x$.

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Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{x/2}$, below by the curve $y = e^{-x/2}$, and on the right by the line $x = 2 \ln 2$.

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Find the area of the region in the first quadrant bounded on the left by the $y$-axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y - 1)^2$, and above right by the line $x = 3 - y$.