Lecture_Template

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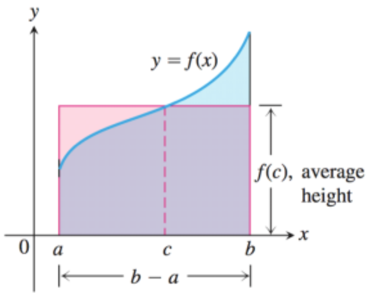
5.4 The Fundamental Theorem of Calculus

Mean Value Theorem for Definite Integrals

THEOREM 3—The Mean Value Theorem for Definite Integrals If f is continuous on $[a, b]$, then at some point c in $[a, b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$



THEOREM 4—The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is $f(x)$:

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x). \qquad (2)$$

Ex: Find dy/dx

$$y = \int_0^x \sqrt{1+t^2}\,dt$$

$$y = \int_{\sqrt{x}}^0 \sin(t^2)\,dt$$

$$y = \left(\int_0^x (t^3+1)^{10}\,dt\right)^3$$

$$y = \int_{2^x}^{e^{x^2}} \frac{1}{\sqrt{t}}\,dt$$

Ex: Determine $f(5)$ if f is cont and

$$\int_0^{4x+\sin(\pi x)} f(t)\,dt = x^2 \text{ for all } x.$$

THEOREM 4 (Continued)—The Fundamental Theorem of Calculus, Part 2 If f is continuous at every point in $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Ex: $\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt$

$$\int_0^{1/\sqrt{3}} \frac{dx}{1 + 4x^2}$$

Determine $f(5)$ if $\int_0^{f(x)} t^2\,dt = 4x + \sin(\pi x)$

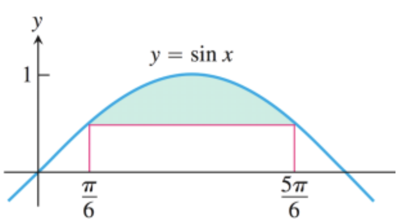
Area

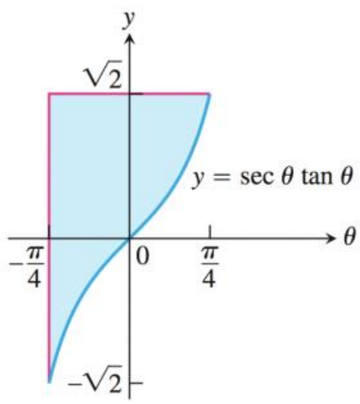
Summary:

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$:

1. Subdivide $[a, b]$ at the zeros of f.
2. Integrate f over each subinterval.
3. Add the absolute values of the integrals.

Ex: Find the areas of the shaded regions





Ex: find the total area between the region and the x-axis.

$$y = -x^2 - 2x, \quad -3 \le x \le 2$$

5.5 Indefinite Integrals and the Substitution Method

THEOREM 6—The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I, and f is continuous on I, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

Ex: $$\int x \sin(2x^2)\,dx, \quad u = 2x^2$$

$$\int \sec 2t \tan 2t\,dt, \quad u = 2t$$

$$\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\,dt$$

$$\int \frac{(1 + \sqrt{x})^{1/3}}{\sqrt{x}}\,dx$$

$$\int \frac{1}{\sqrt{5s + 4}}\,ds$$

$$\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}\,d\theta$$

$$\int \sqrt{\frac{x-1}{x^5}}\,dx$$

The Integrals of $\sin^2 x$ and $\cos^2 x$

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \sin^2 x\,dx$$

$$\int \cos^2 x\,dx$$

5.6 Substitution and Area Between Curves

THEOREM 7—Substitution in Definite Integrals If g' is continuous on the interval $[a, b]$ and f is continuous on the range of $g(x) = u$, then

$$\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

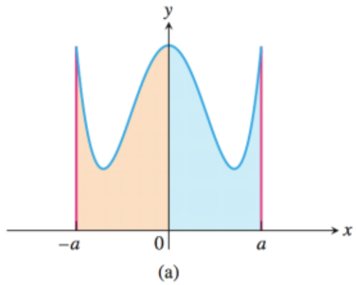
Ex: $$\int_2^{16} \frac{dx}{2x\sqrt{\ln x}}$$

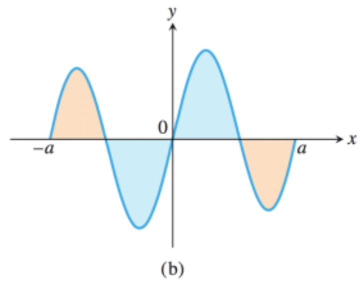
Definite Integrals of Symmetric Functions

THEOREM 8 Let f be continuous on the symmetric interval $[-a, a]$.

(a) If f is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

(b) If f is odd, then $\int_{-a}^{a} f(x)\,dx = 0$.

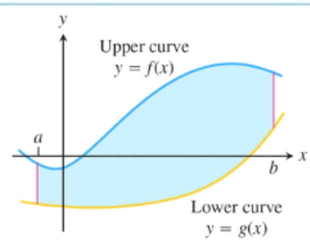




Areas Between Curves

DEFINITION If f and g are continuous with $f(x) \geq g(x)$ throughout $[a, b]$, then the **area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b** is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]\, dx.$$



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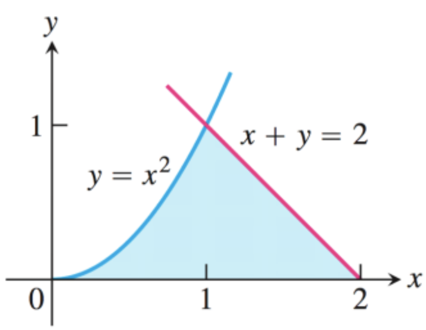

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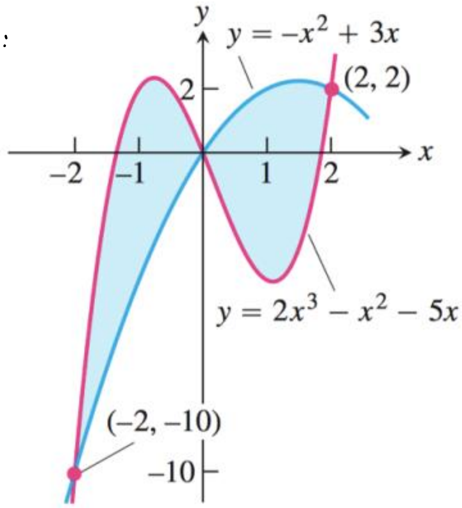
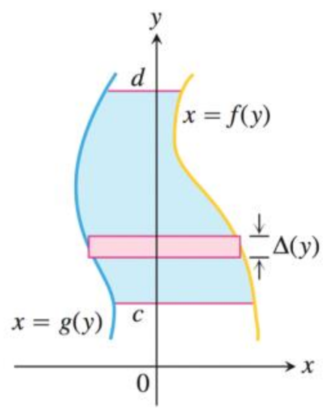


Integration with Respect to y



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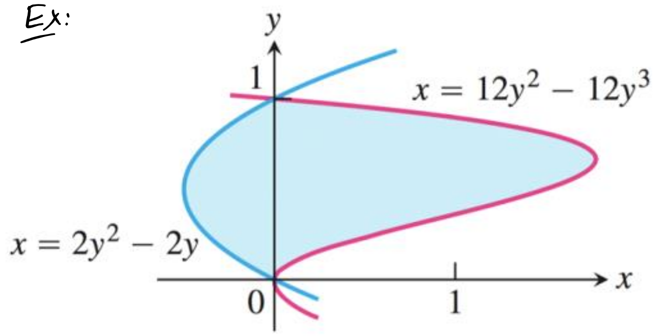

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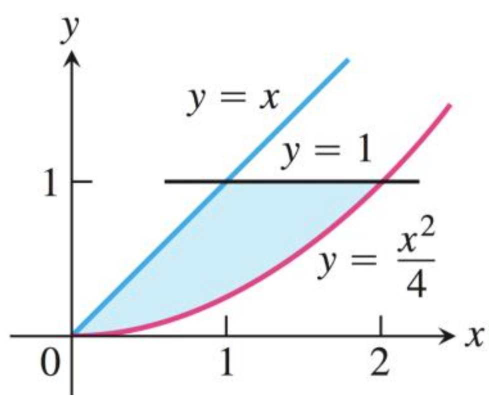

Ex: Find the areas of the regions enclosed by the curves

- $y^2 - 4x = 4$ and $4x - y = 16$
- $x = y^3 - y^2$ and $x = 2y$
- $y = \sqrt{|x|}$ and $5y = x + 6$
- $y = 8\cos x$ and $y = \sec^2 x, \quad -\pi/3 \le x \le \pi/3$