Problem Solving_Template

2.1 Rates of Change

Ex) find the average rate of change of the function

1) $g(t) = 2 + \cos t$

a. $[0, \pi]$ **b.** $[-\pi, \pi]$

2) $R(\theta) = \sqrt{4\theta + 1}$; $[0, 2]$

Ex) Let $f(t) = 1/t$ for $t \neq 0$.

a. Find the average rate of change of f with respect to t over the intervals (i) from $t = 2$ to $t = 3$, and (ii) from $t = 2$ to $t = T$.

2.2 Limit of a Function and Limit Laws

Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?

a. $\lim_{x\to 0} f(x)$ exists.

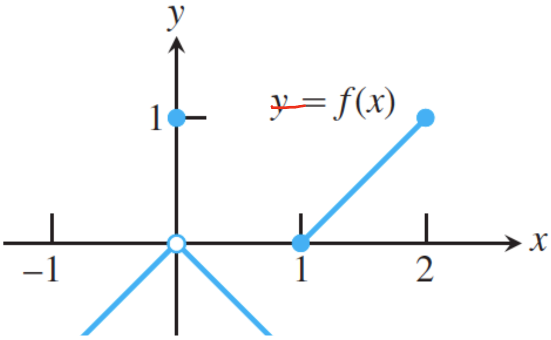
b. $\lim_{x\to 0} f(x) = 0$

c. $\lim_{x\to 0} f(x) = 1$

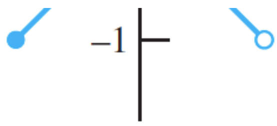
d. $\lim_{x\to 1} f(x) = 1$

e. $\lim_{x\to 1} f(x) = 0$

f. $\lim_{x\to x_0} f(x)$ exists at every point x_0 in $(-1, 1)$



g. $\lim_{x \to 1} f(x)$ does not exist.



Ex If $\lim_{x \to 1} f(x) = 5$, must f be defined at $x = 1$? If it is, must $f(1) = 5$? Can we conclude *anything* about the values of f at $x = 1$? Explain.

If $f(1) = 5$, must $\lim_{x \to 1} f(x)$ exist? If it does, then must $\lim_{x \to 1} f(x) = 5$? Can we conclude *anything* about $\lim_{x \to 1} f(x)$? Explain.

Ex: $$\lim_{x \to -3} \frac{x + 3}{x^2 + 4x + 3}$$

$$\lim \quad \sqrt{5h + 4} - 2$$

$$\lim_{h\to 0}\frac{\qquad}{h}$$

Ex: $\displaystyle\lim_{x\to 0}\frac{1+x+\sin x}{3\cos x}$

Ex: $\displaystyle\lim_{x\to 2}\frac{\sqrt{x^2+12}-4}{x-2}$

Suppose that $\lim_{x\to -2} p(x) = 4$, $\lim_{x\to -2} r(x) = 0$, and $\lim_{x\to -2} s(x) = -3$. Find

a. $\displaystyle\lim_{x\to -2}(p(x)+r(x)+s(x))$

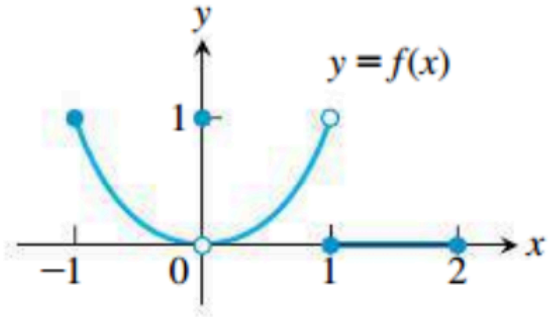
b. $\displaystyle\lim_{x\to -2} p(x)\cdot r(x)\cdot s(x)$

c. $\displaystyle\lim_{x\to -2}(-4p(x)+5r(x))/s(x)$

2.4 One-Sided Limits

Finding Limits Graphically

1. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



a. $\lim_{x \to -1^+} f(x) = 1$

b. $\lim_{x \to 0^-} f(x) = 0$

c. $\lim_{x \to 0^-} f(x) = 1$

d. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

e. $\lim_{x \to 0} f(x)$ exists.

f. $\lim_{x \to 0} f(x) = 0$

g. $\lim_{x \to 0} f(x) = 1$

h. $\lim_{x \to 1} f(x) = 1$

i. $\lim_{x \to 1} f(x) = 0$

j. $\lim_{x \to 2^-} f(x) = 2$

k. $\lim_{x \to -1^-} f(x)$ does not exist.

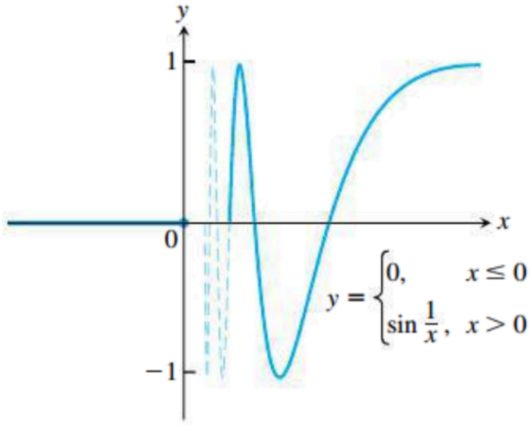
l. $\lim_{x \to 2^+} f(x) = 0$

In Exercises 5 and 6, explain why the limits do not exist.

5. $\lim_{x \to 0} \frac{x}{|x|}$

6. $\lim_{x \to 1} \frac{1}{x - 1}$

5. Let $f(x) = \begin{cases} 0, & x \le 0 \\ \sin\dfrac{1}{x}, & x > 0. \end{cases}$



a. Does $\lim_{x\to 0^+} f(x)$ exist? If so, what is it? If not, why not?

b. Does $\lim_{x\to 0^-} f(x)$ exist? If so, what is it? If not, why not?

c. Does $\lim_{x\to 0} f(x)$ exist? If so, what is it? If not, why not?

Finding One-Sided Limits Algebraically

Find the limits in Exercises 11–20.

15. $\displaystyle\lim_{h\to 0^+} \frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$

16. $\displaystyle\lim_{h\to 0^-} \frac{\sqrt{6}-\sqrt{5h^2+11h+6}}{h}$

17. a. $\displaystyle\lim_{x\to -2^+} (x+3)\frac{|x+2|}{x+2}$ **b.** $\displaystyle\lim_{x\to -2^-} (x+3)\frac{|x+2|}{x+2}$

18. a. $\displaystyle\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$ **b.** $\displaystyle\lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$

19. **a.** $\lim_{x \to 0^+} \frac{|\sin x|}{\sin x}$ **b.** $\lim_{x \to 0^-} \frac{|\sin x|}{\sin x}$

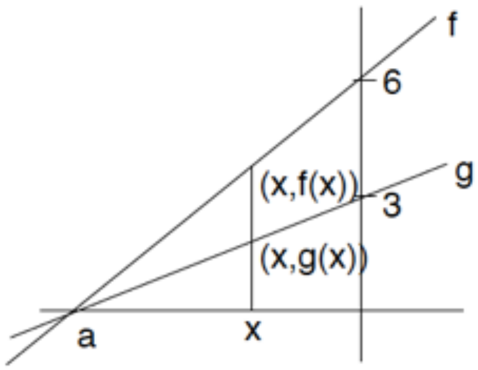
20. **a.** $\lim_{x \to 0^+} \frac{1 - \cos x}{|\cos x - 1|}$ **b.** $\lim_{x \to 0^-} \frac{\cos x - 1}{|\cos x - 1|}$

Ex If $\lim_{x \to 0} \frac{f(x)}{x^2} = 1$, find

a. $\lim_{x \to 0} f(x)$ **b.** $\lim_{x \to 0} \frac{f(x)}{x}$

Ex: If $\lim_{x \to 2} \frac{f(x) - 5}{x - 2} = 3$, find $\lim_{x \to 2} f(x)$.

Ex [D] Suppose you have two linear functions f and g shown below.



Then $\lim_{x \to a} \dfrac{f(x)}{g(x)}$ is

(a) 2

(b) does not exist

(c) not enough information

(d) 3

Ex It can be shown that the inequalities

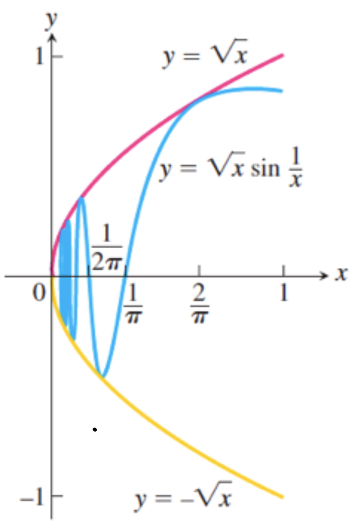
$$1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2\cos x} < 1$$

hold for all values of x close to zero. What, if anything, does this tell you about

$$\lim_{x \to 0} \frac{x \sin x}{2 - 2\cos x}?$$

Give reasons for your answer.

Let $g(x) = \sqrt{x}\sin(1/x)$.



$= g(x)$.

a. Does $\lim_{x \to 0^+} g(x)$ exist? If so, what is it? If not, why not?

b. Does $\lim_{x \to 0^-} g(x)$ exist? If so, what is it? If not, why not?

c. Does $\lim_{x \to 0} g(x)$ exist? If so, what is it? If not, why not?

Using $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

Find the limits in Exercises 21–42.

21. $\lim_{\theta \to 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$

22. $\lim_{t \to 0} \frac{\sin kt}{t}$ (k constant)

23. $\lim_{y \to 0} \frac{\sin 3y}{4y}$

24. $\lim_{h \to 0^-} \frac{h}{\sin 3h}$

25. $\lim_{x \to 0} \frac{\tan 2x}{x}$

26. $\lim_{t \to 0} \frac{2t}{\tan t}$

27. $\lim_{x \to 0} \frac{x \csc 2x}{\cos 5x}$

28. $\lim_{x \to 0} 6x^2(\cot x)(\csc 2x)$

29. $\lim_{x \to 0} \frac{x + x \cos x}{\sin x \cos x}$

30. $\lim_{x \to 0} \frac{x^2 - x + \sin x}{2x}$

31. $\lim_{\theta \to 0} \frac{1 - \cos \theta}{\sin 2\theta}$

32. $\lim_{x \to 0} \frac{x - x \cos x}{\sin^2 3x}$

33. $\lim_{t \to 0} \frac{\sin(1 - \cos t)}{1 - \cos t}$

34. $\lim_{h \to 0} \frac{\sin(\sin h)}{\sin h}$

35. $\lim_{\theta \to 0} \frac{\sin \theta}{\sin 2\theta}$

36. $\lim_{x \to 0} \frac{\sin 5x}{\sin 4x}$

37. $\lim_{\theta \to 0} \theta \cos \theta$

38. $\lim_{\theta \to 0} \sin \theta \cot 2\theta$

39. $\lim_{x \to 0} \frac{\tan 3x}{\sin 8x}$

40. $\lim_{y \to 0} \frac{\sin 3y \cot 5y}{y \cot 4y}$